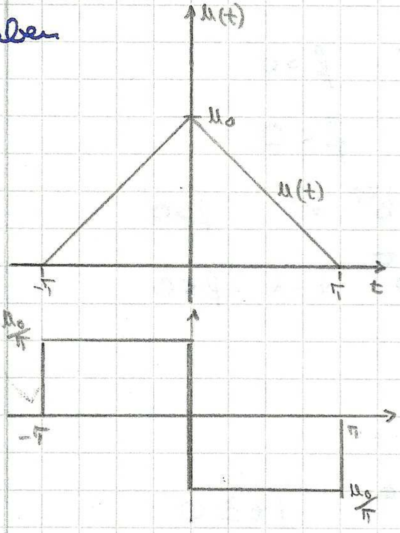
Übungsaufgaben

.3.1



$$u_1(t) = \frac{u_0}{T}\left[\mathrm{rect}_T\left(t+\frac{T}{2}\right) - \mathrm{rect}_T\left(t-\frac{T}{2}\right)\right]$$

$$= \int_{-\infty}^{+\infty} u_1(\tau)\, d\tau$$

$$U(f) = \left(\frac{1}{j2\pi f} + \frac{1}{2}\delta(f)\right) \mathcal{F}\{u_1(t)\}$$

$$= \frac{u_0}{T} \underbrace{\left(\frac{1}{j2\pi f} + \frac{1}{2}\delta(f)\right)}_{a} \left[e^{j2\pi f\frac{T}{2}}\left(T\,\mathrm{si}(\pi f T)\right) - e^{-j2\pi f\frac{T}{2}} T\,\mathrm{si}(\pi f T)\right]$$

$$= a\, u_0\, \mathrm{si}(\pi f T) \underbrace{\left[e^{jT\pi f} - e^{-jT\pi f}\right]}_{\sin(T\pi f)} = a\, 2j\, u_0\, \mathrm{si}(\pi f T)\, \sin(T\pi f)$$

$$= \left[\frac{1}{j2\pi f} + \frac{1}{2}\delta(f)\right] 2j\, u_0 \pi f T\, \mathrm{si}^2(\pi f T)$$

$$= u_0 T\, \mathrm{si}^2(\pi f T) + j \underbrace{\delta(f)\, u_0 \pi f T\, \mathrm{si}^2(\pi f T)}_{0}$$

⇨ andere Variante = Faltungssatz (Δ ist entstanden aus Rechteck in der Faltung mit sich selbst)

.3.2 → Ähnlichkeitssatz

Herleiten kommt nicht in der Prüfung dran

$$u(t) = u_1(at) \qquad u_1(t) \circ\!\!-\!\!\bullet\, U_1(f)$$

$\frac{\tau}{a} = t = -a\infty$

$$U(f) = \int_{-\infty}^{\infty} u(t)\, e^{-j2\pi f t}\, dt = \int_{-\infty}^{\infty} u_1(at)\, e^{-j2\pi f t}\, dt \qquad \text{→ Substitution: } \tau = at$$

$$t = \frac{\tau}{a}$$

$$= \int_{-\infty a}^{a\infty} u_1(\tau)\, e^{-j2\pi f \frac{\tau}{a}} \frac{1}{a}\, d\tau = \frac{1}{a}\int_{-a\infty}^{a\infty} u_1(\tau)\, e^{-2\pi \frac{f}{a}\tau}\, d\tau$$

→ Substitution: $\varphi = \frac{f}{a}$

$$= \underbrace{\frac{1}{a}\int_{-a\infty}^{a\infty} u_1(\tau)\cdot e^{-2\pi\varphi\tau}\, d\tau}_{= \frac{1}{a}\begin{cases} U_1(\varphi) & a>0 \\ -U_1(\varphi) & a<0 \end{cases}} = \frac{1}{|a|} U_1(\varphi)$$

$$U(f) = \frac{1}{|a|} U_1\left(\frac{f}{a}\right)$$

I.3.3 Phasensprung

$$U_2(f) = U_1(f)\, e^{j\varphi(f)} \qquad \varphi(f) = \begin{cases} -\pi & f>0 \\ \pi & f<0 \end{cases}$$

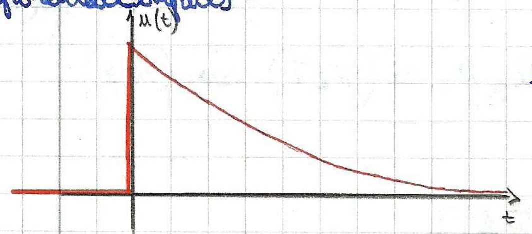
! Nicht der Verschiebungssatz !

$$e^{j\varphi(f)} = \begin{cases} e^{-j\pi} = -1 & f>0 \\ e^{j\pi} = -1 & f<0 \end{cases}$$

$$U_2(f) = -U_1(f)$$

$$u_2(t) = -u_1(t)$$

I.3.4 Exponentialimpuls



$$u(t) = \begin{cases} 2V e^{-\sigma t} & \mid t \geq 0 \\ 0 & \mid t<0 \end{cases} \quad \text{mit } \sigma > 0$$

a) $E = \int_{-\infty}^{\infty} |u(t)|^2 dt = \int_0^{\infty} 4V^2 e^{-2\sigma t} dt = -\frac{4V^2}{2\sigma} e^{-2\sigma t} \Big|_0^{\infty} = \frac{4V^2}{2\sigma} < \infty$ → Energiesignal

b) $U(f) = \int_{-\infty}^{\infty} u(t)\, e^{-j2\pi f t} dt = \int_0^{\infty} 2V e^{-\sigma t} e^{-j2\pi f t} dt$

$$= 2V \int_0^{\infty} e^{-(\sigma + j2\pi f)t} dt = 2V \left[\frac{1}{-(\sigma + j2\pi f)} e^{-(\sigma + j2\pi f)t} \right]_0^{\infty}$$

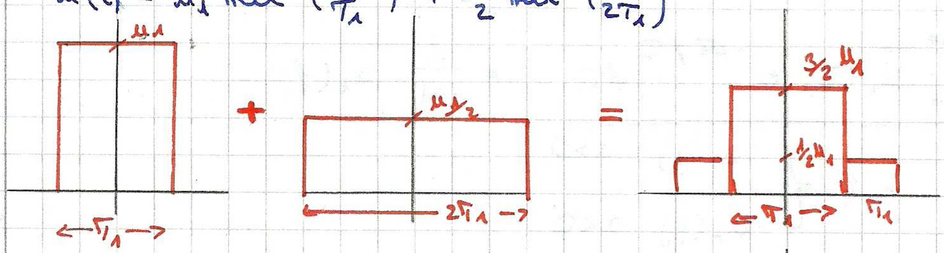
$$= \frac{2V}{\sigma + j2\pi f}$$

I.3.5 si-Funktionen

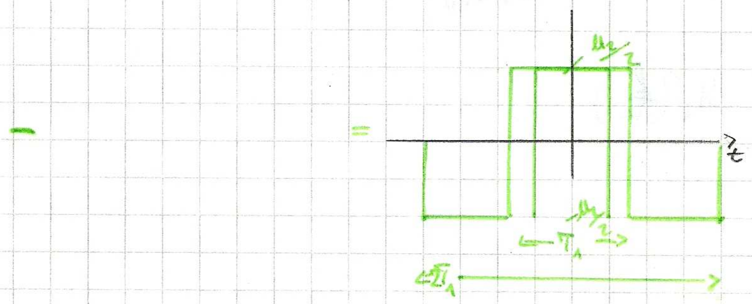
a) $U(f) = U_1 T_1\, si(\pi T_1 f) + U_2 T_2\, si(\pi T_2 f)$; $U_2 = \frac{U_1}{2}$; $T_2 = 2T_1$

$$U(f) = U_1 T_1\, si(\pi T_1 f) + \frac{U_1}{2}\, 2T_1\, si(\pi\, 2T_1 f)$$

$$u(t) = U_1\, rect\left(\frac{t}{T_1}\right) + \frac{U_1}{2}\, rect\left(\frac{t}{2T_1}\right)$$



b)

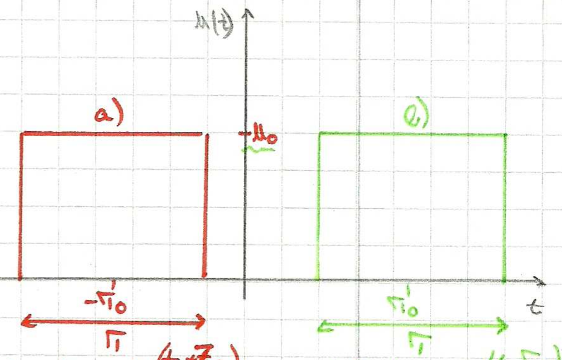


.3.6 Rechteckimpuls

a) $U(f) = U_0 T \; si(\pi T f) \; e^{j 2\pi T_0 f}$

b) $U(f) = U_0 T \; si(\pi T f) \; e^{-j 2\pi T_0 f}$

Verschiebungssatz: $u(t-t_0) \circ\!\!-\!\!\bullet \; U(f)\, e^{-j 2\pi T_0 f}$



a) $u(t) = U_0 \, rect\left(\frac{t+T_0}{T}\right)$

b) $u(t) = U_0 \, rect\left(\frac{t-T_0}{T}\right)$

.3.7 Symmetrische Aufspaltung

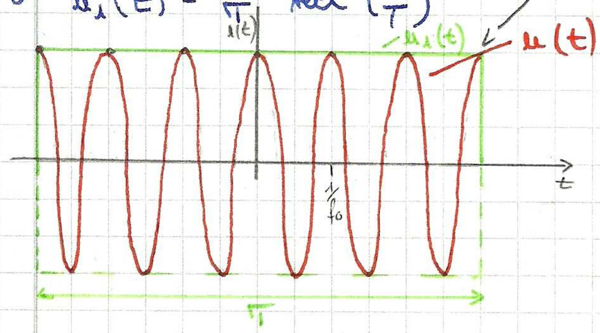
$$U(f) = \tfrac{1}{2} U_1(f-f_0) + \tfrac{1}{2} U_1(f+f_0)$$

$U_1(f) = C \cdot si(\pi T f)$ $\qquad f_0 \gg \frac{1}{T}$

—> sonst eventuell nur Cosinusausschnitt

1. Lösungsmögl ⇒ Symmetrische Aufspaltung: $U(f) \bullet\!\!-\!\!\circ \; u_1(t) \cos 2\pi f_0 t$

$u_1(t) = \frac{C}{T} \; rect\left(\frac{t}{T}\right)$



2. Lösungsmögl ⇒ Verschiebungssatz:

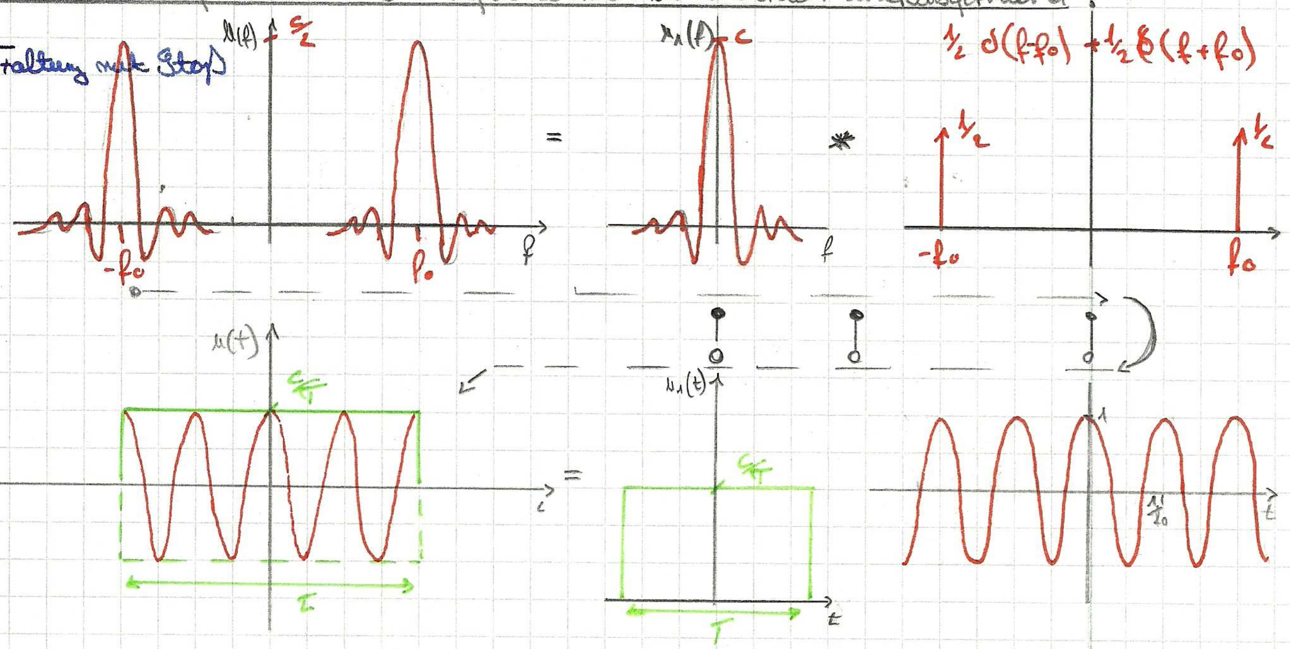
$U(f) = \frac{1}{2} U_1(f-f_0) + \frac{1}{2} U_1(f+f_0)$

$u(t) = \frac{1}{2} u_1(t)\, e^{j 2\pi f_0 t} + \frac{1}{2} u_1(t)\, e^{-j 2\pi f_0 t} = u_1(t) \cdot \underbrace{\tfrac{1}{2}\left(e^{j 2\pi f_0 t} + e^{-j 2\pi f_0 t}\right)}$

$= u_1(t) \qquad \cos 2\pi f_0 t$

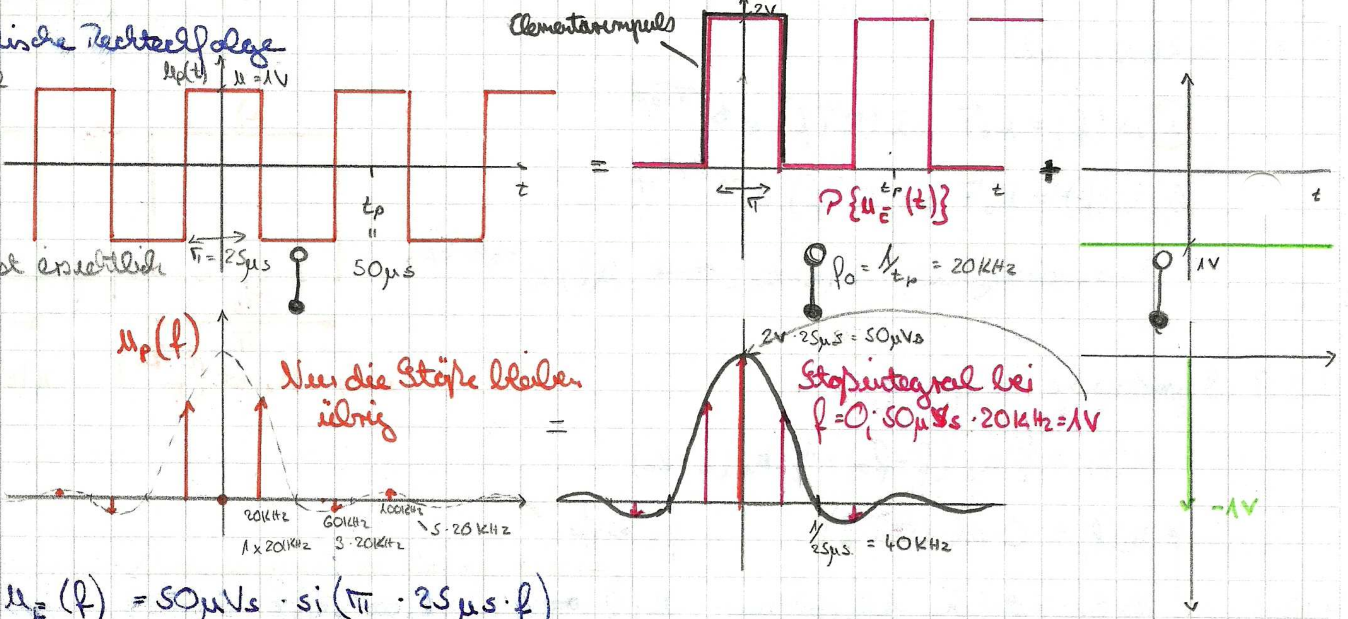
! Inverse Fouriertransformation = Vom Frequenzbereich in Zeitbereich zu transformieren !

3. Lösungsmöglichkeiten ⇒ Faltung mit Stoß



I.3.8 Periodische Rechteckfolge

Für die Prüfung egal ob mathematisch oder graphische Lösung
→ Hauptsache der Lösungsweg ist ersichtlich



$$u_E(f) = 50\mu Vs \cdot si(\pi \cdot 25\mu s \cdot f)$$

$$A\{u(f)\} = \sum_{\mu=-\infty}^{\infty} f_0\, u_E(\mu f_0)\,\delta(f-\mu f_0) = \sum_{\mu=-\infty}^{\infty} 1V\, si(\pi\, 25\mu s \cdot \mu \cdot 20kHz)\cdot \delta(f-\mu \cdot 20kHz)$$

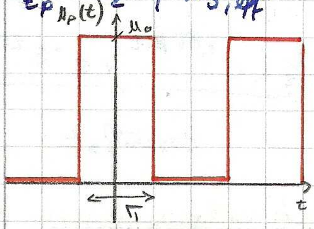
$$= \sum_{\mu=-\infty}^{\infty} 1V\, si\left(\mu \cdot \frac{\pi}{2}\right)\delta(f-\mu f_0)$$

andere Schreibweise

$$\Rightarrow U_p(f) = \sum_{\substack{\mu=-\infty \\ \mu\neq 0}}^{\infty} 1V\, si\left(\mu\frac{\pi}{2}\right)\delta(f-\mu\, 20kHz) = \sum_{\mu=-\infty}^{\infty}\left[1V\, si\left(\mu\frac{\pi}{2}\right)\delta(f-\mu\, 20kHz)\right] - 1V\delta(f)$$

I.3.9 Periodische Rechteckfolge

$\frac{T}{t_p} = \frac{1}{2}$; $U_{3,eff} = 100mV$

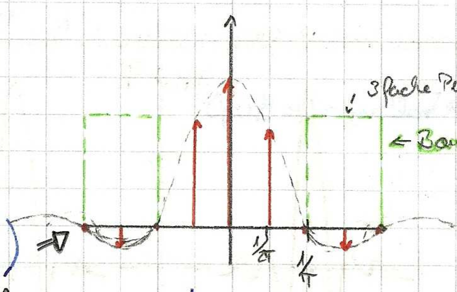


$$u_p(t) = P\{u(t)\} \quad ; \quad u(t) = U_0\, rect\left(\frac{t}{T}\right) = U_0\, rect_T(t)$$

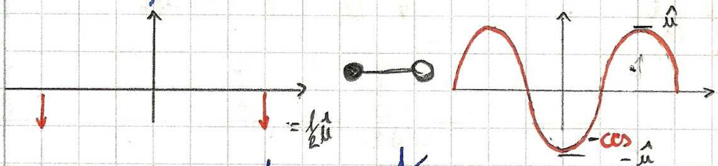
$$U_p(f) = A\{U(f)\} = \sum_{\mu=-\infty}^{\infty} \frac{1}{t_p}\, U\left(\mu\frac{1}{t_p}\right)\delta\left(f-\mu\frac{1}{t_p}\right)$$

$$U(f) = U_0 T\, si(\pi T f)$$

$$U\left(\mu\frac{1}{t_p}\right) = U_0 T\, si\left(\frac{1}{2}\pi\mu\right) \qquad t_p = 2T$$



$$U_p(f) = \sum_{\mu=-\infty}^{\infty} \frac{1}{2} U_0\, si\left(\frac{1}{2}\pi\mu\right)\delta\left(f-\mu\frac{1}{2T}\right)$$



$f_{cos} = 3\frac{1}{2T}$ $\Rightarrow \mu = 3; -3$

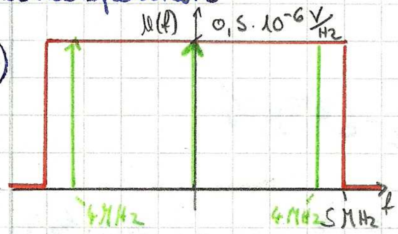
für $\mu = 3$ $\Rightarrow \frac{1}{2}\hat{u} = \frac{1}{3\pi} U_0$

$$\frac{1}{2}u_{eff}\sqrt{2} = \frac{1}{3\pi}U_0 \Rightarrow U_0 = \frac{1}{2}u_{eff}\cdot\sqrt{2}\, 3\pi = 0{,}66V$$

3.10 Rechteckspektrum

$11\ 14^{25}$

b)



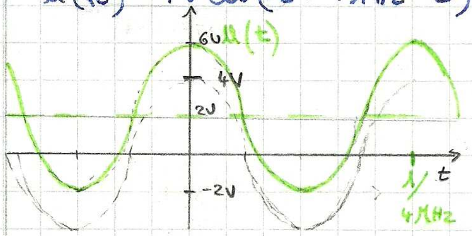
$P\{u(t)\} \circ\!\!-\!\!\bullet A\{U(f)\}$ $\qquad f_0 = \frac{1}{t_p}$

b) $f_0 = \frac{1}{0{,}25\mu s} = 4MHz$

$A\{U(f)\} = \underbrace{0{,}5 \cdot 10^{-6}\frac{V}{Hz}\cdot 4MHz}_{2V}\left(\delta(f+4MHz)+\delta(f-4MHz)+\delta(f)\right)$

$\hat{u}\cos 2\pi f_0 t \circ\!\!-\!\!\bullet \frac{\hat{u}}{2}\left[\delta(f-f_0)+\delta(f+f_0)\right]$

$u(t) = 4V\cos(2\pi\, 4MHz\cdot t) + 2V$



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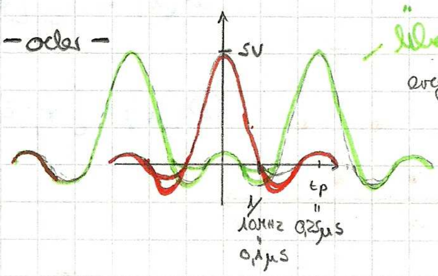
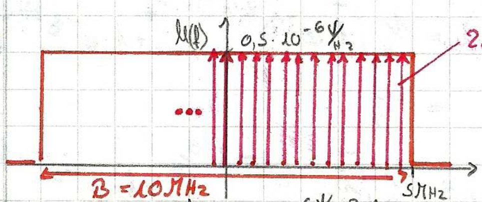


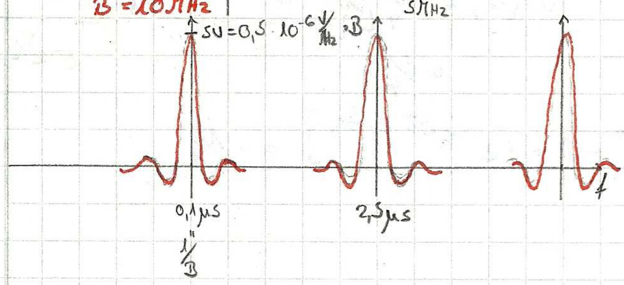
Überlagerung der si-Fkten ergibt die gleiche Fkt

a)

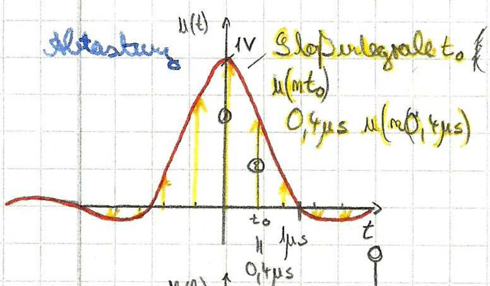


25 Stöße Stoßintegral $0{,}4MHz \cdot 0{,}5\cdot 10^{-6}\,V/Hz = 0{,}2V$

$t_p = 2{,}5\mu s \qquad f_0 = \frac{1}{t_p} = 0{,}4MHz$



3.11 Abtastung



Stoßintegrale $t_0 \cdot u(mt_0)$

$0{,}4\mu s \cdot u(m\,0{,}4\mu s)$

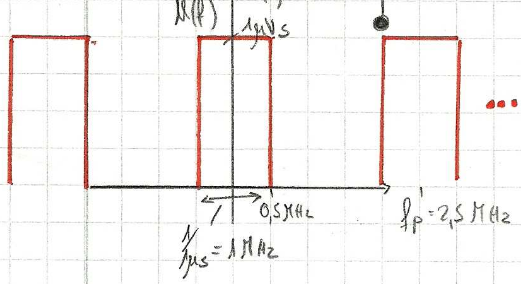
$f_p = 2{,}5MHz \qquad t_0 = \frac{1}{f_p} = 0{,}4\mu s \qquad u(t) = 1V \cdot si(\pi\, 1MHz\, t)$

$A\{u(t)\} = u(t)\cdot ш\left(\frac{t}{t_0}\right) = t_0 \sum_{m=-\infty}^{\infty} u(mt_0)\,\delta(t-mt_0)$

Werte: ① $0{,}4\mu s \cdot 1V = 0{,}4\mu Vs$ ② $0{,}4\mu s\; si(\pi\, 1MHz \cdot 0{,}4\mu s) = 0{,}4\mu s\; si(0{,}4\pi)$

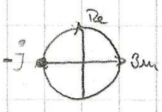
$P\{U(f)\} = U(f) * ш\left(\frac{f}{f_0}\right) = \sum_{r=-\infty}^{\infty} U(f - r f_p)$

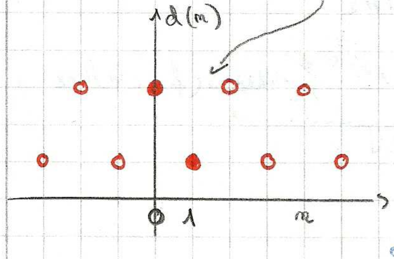
bzw. $U(f) = 1\mu Vs\; rect\left(\frac{f}{1MHz}\right)$



I.3.12 DFT

I 3.64 a) $d(n) = \{6;2\}$ N=2

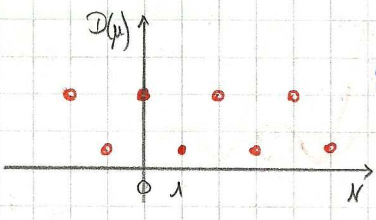




$$D(\mu) = \frac{1}{N}\sum_{n=0}^{N-1} d(n)\, e^{-j2\pi\mu\, n/N}$$

$$D(0) = \frac{1}{2}\left[6\cdot e^{-j2\pi\cdot 0\cdot 0/2} + 2e^{-j2\pi 0\cdot 1/2}\right] = 4$$

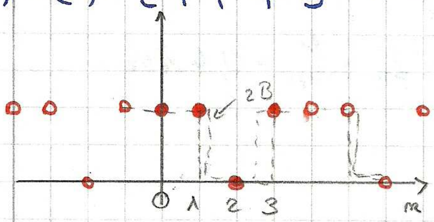
$$D(1) = \frac{1}{2}\left[6\cdot e^{-j2\pi 1\cdot 0/2} + 2\cdot \underbrace{e^{-j2\pi 1\cdot 1/2}}_{e^{-j\pi}=-1}\right] = 2$$



$$D(\mu) = \{4;2\}$$

b) $d(n) = \{1;1;0;1\}$ $N=4$ $\circ\!\xrightarrow{DFT}\!\bullet$ $D(0) = \frac{1}{4}\left[1\cdot\underbrace{e^{-j2\pi 0\cdot 0/4}}_{e^0} + 1\cdot e^{-j2\pi 0\cdot 1/4} + 0 + 1e^{-j2\pi 0\cdot 3/4}\right.$

$= 3/4$



$$D(1) = \frac{1}{4}\left[1\underbrace{e^{-j2\pi 1\cdot 0/4}}_{e^0} + 1\cdot\underbrace{e^{-j2\pi 1\cdot 1/4}}_{e^{-j\pi/2}=-j} + 0 + 1\underbrace{e^{-j2\pi 1\cdot 3/4}}_{e^{-j\frac{3}{2}\pi}=j}\right]$$

$= 1/4$

$$D(2) = \frac{1}{4}\left[1\cdot e^0 + 1\cdot\underbrace{e^{-j2\pi 2\cdot 1/4}}_{=-1} + 0 + 1\underbrace{e^{-j2\pi 2\cdot 3/4}}_{=-1}\right] = -1/4$$

$D(3) = 1/4$

$$\Rightarrow D(\mu) = \{3/4;\ 1/4;\ -1/4;\ 1/4\}$$