II.2.1

a) $U_2(t) = U_R(t)$

$$i_C(t) = C \cdot \dot{U}_2(t) \quad ; \quad \frac{i_C}{C} = \dot{U}_C(t) \rightarrow \int \frac{i_C(t)}{C} dt = U_C(t)$$

$$U_1(t) = U_C(t) + U_2(t)$$

$$U_1(t) = \int \frac{i_C(t)}{C} + U_2(t)$$

$$\dot{U}_1(t) = \frac{i_C(t)}{C} + \dot{U}_2(t) \quad ; \quad i_C(t) = i_R(t) \; ; \; i_R(t) = \frac{U_2(t)}{R}$$

$$\dot{U}_1(t) = \frac{U_2(t)}{RC} + \dot{U}_2(t)$$

b)

$$U_1(f) \cdot j2\pi f = U_2(f) \cdot \frac{1}{RC} + U_2(f) \cdot j2\pi f$$

$$\frac{U_2(f)}{U_1(f)} = \frac{j2\pi f}{\frac{1}{RC} + j2\pi f} = \frac{RC \cdot j2\pi f}{1 + RCj2\pi f} = \frac{j2\pi f \tau_1}{1 + j2\pi f \tau_1}$$

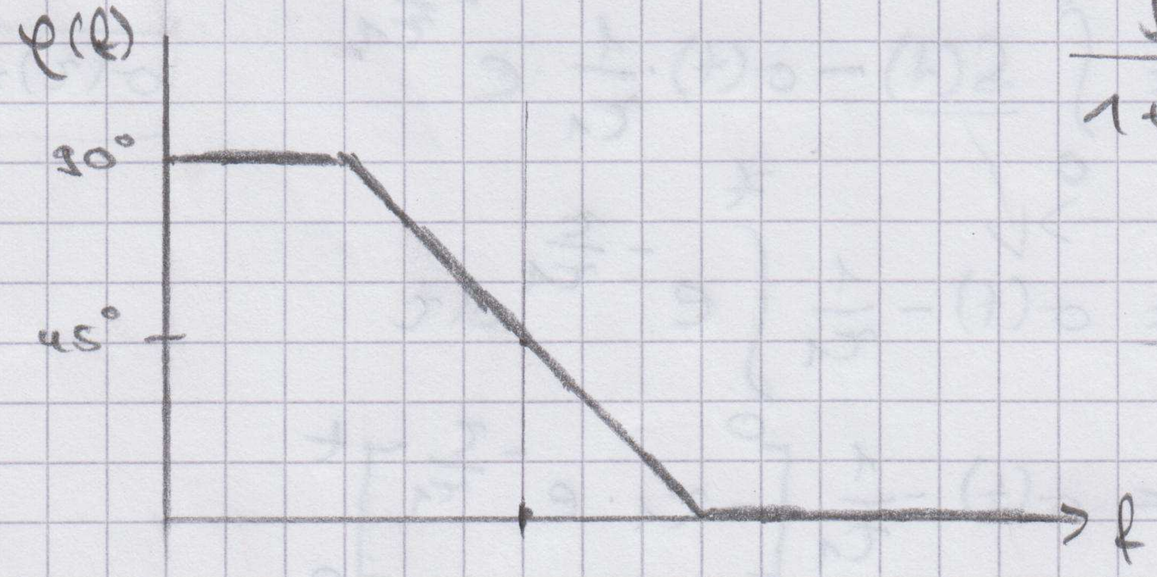
$$\frac{1}{2\pi\tau_1} = 1591\,Hz$$

|a|

Verlauf in Tabelle nachschauen

$$\frac{j\frac{\omega}{\omega_P}}{1 + j\frac{\omega}{\omega_P}}$$

$\varphi(f)$



c) $H(f) = j2\pi f \cdot \tau_1 \cdot \frac{1}{1+j2\pi f\tau_1}$

Einteilung für Transformation

Differenziationssatz (F-9)

Skript II.2-5

$$\frac{d}{dt}\left(\tau_1 \cdot \frac{1}{\tau_1} \cdot \sigma(t) \cdot e^{-\frac{t}{\tau_1}}\right)$$

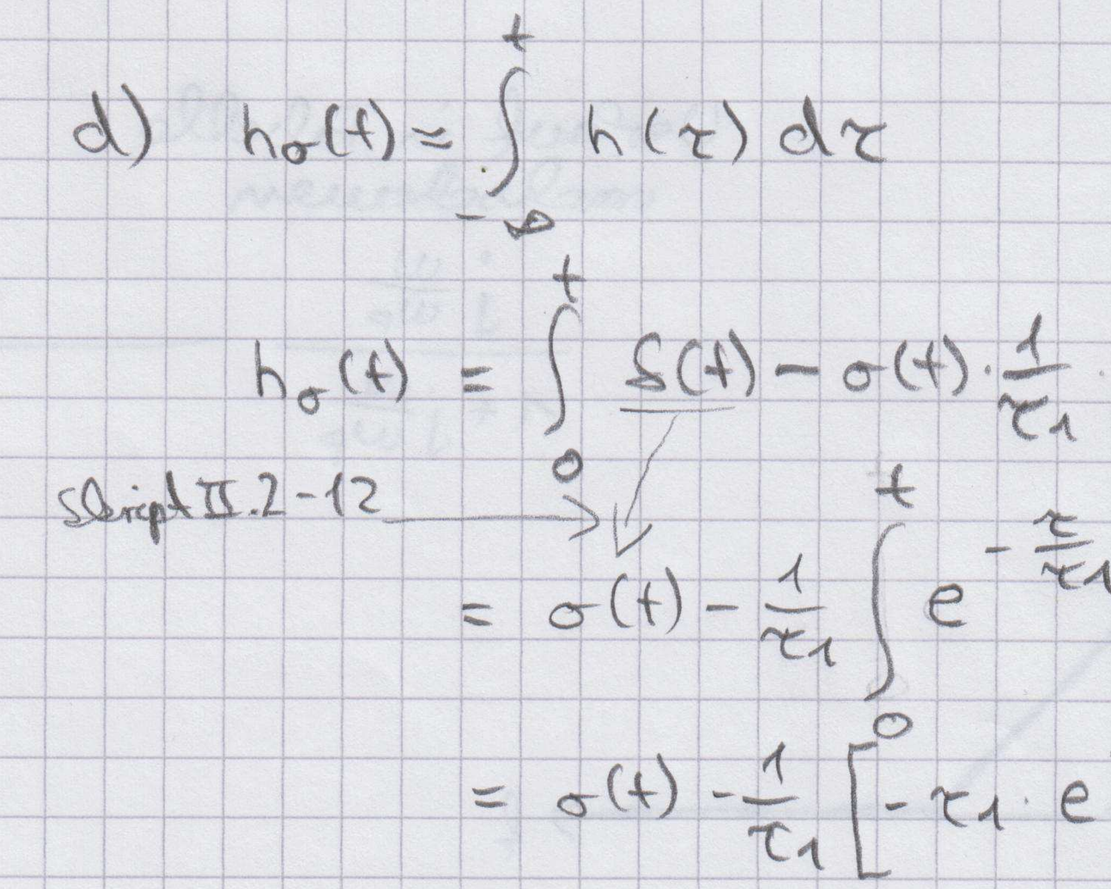
$$= \frac{d}{dt}\left(\sigma(t) \cdot e^{-\frac{t}{\tau_1}}\right)$$

Skript II.2-12

Partielle Ableitung

$$= \underbrace{\delta(t) \cdot e^{-\frac{t}{\tau_1}}}_{\delta(t)} + \sigma(t) \cdot \left(-\frac{1}{\tau_1}\right) \cdot e^{-\frac{t}{\tau_1}}$$

$$h(t) = \delta(t) - \sigma(t) \cdot \frac{1}{\tau_1} \cdot e^{-\frac{t}{\tau_1}}$$



d) $h_\sigma(t) = \int_{-\infty}^{t} h(\tau)\, d\tau$

$$h_\sigma(t) = \int_0^t \underbrace{\delta(t)}_{\text{Skript II.2-12}} - \sigma(t) \cdot \frac{1}{\tau_1} e^{-\frac{\tau}{\tau_1}}$$

$\boxed{\sigma(\tau) = 1,\ t > 0}$

$$= \sigma(t) - \frac{1}{\tau_1}\int_0^t e^{-\frac{\tau}{\tau_1}}\, d\tau$$

$$= \sigma(t) - \frac{1}{\tau_1}\left[-\tau_1 \cdot e^{-\frac{\tau}{\tau_1}}\right]_0^t$$

$$h_\sigma(t) = \sigma(t) - \left(1 - e^{-\frac{t}{\tau_1}}\right)$$