Übungsblatt 3

1) (1)
$$F_1(s) = \frac{\Omega}{U_a} = \frac{\frac{K_1}{1+T_1\cdot s}\cdot\frac{K_3}{s}}{1+\frac{K_1}{1+T_1\cdot s}\cdot\frac{K_3}{s}\cdot K_2} = \frac{K_1\cdot K_3}{T_1 s^2+s+K_1\cdot K_2\cdot K_3}$$

$$= \frac{K\omega_0^2}{s^2+2D\omega_0 s + \omega_0^2} = \frac{\frac{K_1\cdot K_3}{T_1}}{s^2+\frac{1}{T_1}\cdot s+\frac{K_1\cdot K_2\cdot K_3}{T_1}}$$

$$= \frac{\frac{1}{64}}{s^2+\frac{1}{16}s+\frac{1}{256}}$$

$$F_2(s) = \frac{\Omega}{M_D} = -\frac{\frac{K_3}{s}}{1+\frac{K_3}{s}\cdot K_2\cdot\frac{K_1}{1+T_1 s}} = \frac{-K_3(1+T_1\cdot s)}{T_1\cdot s^2+s+K_1\cdot K_2\cdot K_3}$$

$$= \frac{-4(1+16s)}{256s^2+16s+1}$$

(2)
$$\omega_0 = \sqrt{\frac{K_1\cdot K_2\cdot K_3}{T_1}} = \frac{1}{16}$$

$$2\cdot D\cdot\omega_0 = \frac{1}{T_1} \Rightarrow D = \frac{1}{2T_1}\cdot\sqrt{\frac{T_1}{K_1\cdot K_2 K_3}}$$

$$= \frac{1}{2\sqrt{T_1 K_1\cdot K_2 K_3}} = 0{,}5$$

$$\omega_e = \omega_0\cdot\sqrt{1-D^2} = 0{,}054$$

③ $F_1(s): \Omega(t=0^+) = \lim_{s\to\infty} (s \cdot F_1(s) \cdot U_a(s)) = \lim_{s\to\infty} F_1(s) = 0$ (10)

$\quad " \quad : \Omega(t\to\infty) = \lim_{s\to 0} F_1(s) = 4$

$F_2(s): \Omega(t=0^+) = \lim_{s\to\infty} F_2(s) = 0$

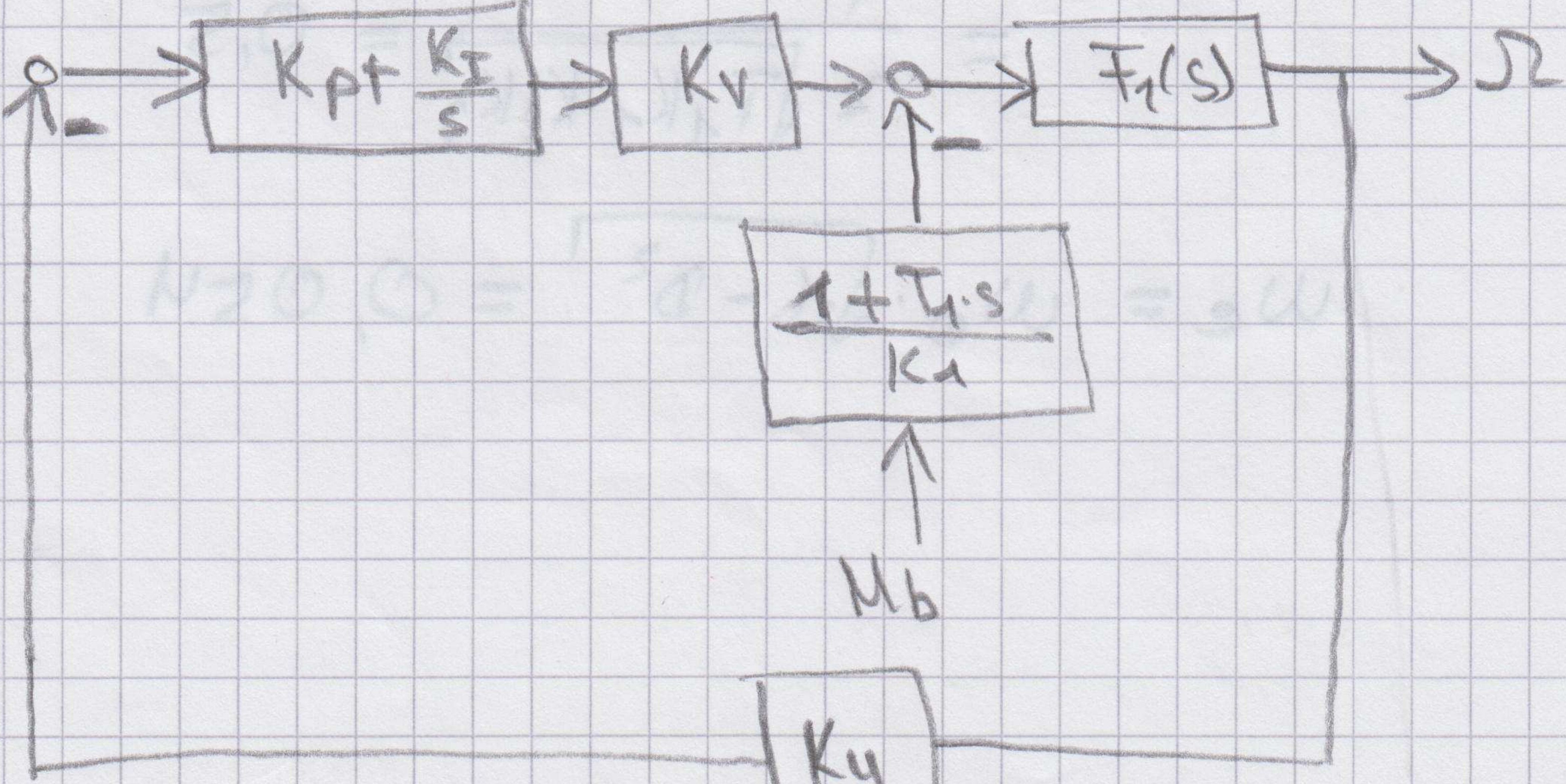
$\quad " \quad : \Omega(t\to\infty) = \lim_{s\to 0} F_2(s) = -4$

④ $F_3(s) = \frac{\Omega}{U_w} = \frac{\left(K_P + \frac{K_I}{s}\right) \cdot K_V \cdot F_1(s)}{1 + \left(K_P + \frac{K_I}{s}\right) K_V \cdot F_1(s) \cdot K_4}$

$= \frac{4(K_P \cdot s + K_I)}{256 s^3 + 16 s^2 + (1 + 4K_P) s + 4K_I}$

$F_4(s) = \frac{\Omega}{M_b} = \frac{-\left(\frac{1 + T_1 \cdot s}{K_1}\right) \cdot F_1(s)}{1 + F_1(s) \cdot K_V K_4 \left(K_P + \frac{K_I}{s}\right)}$

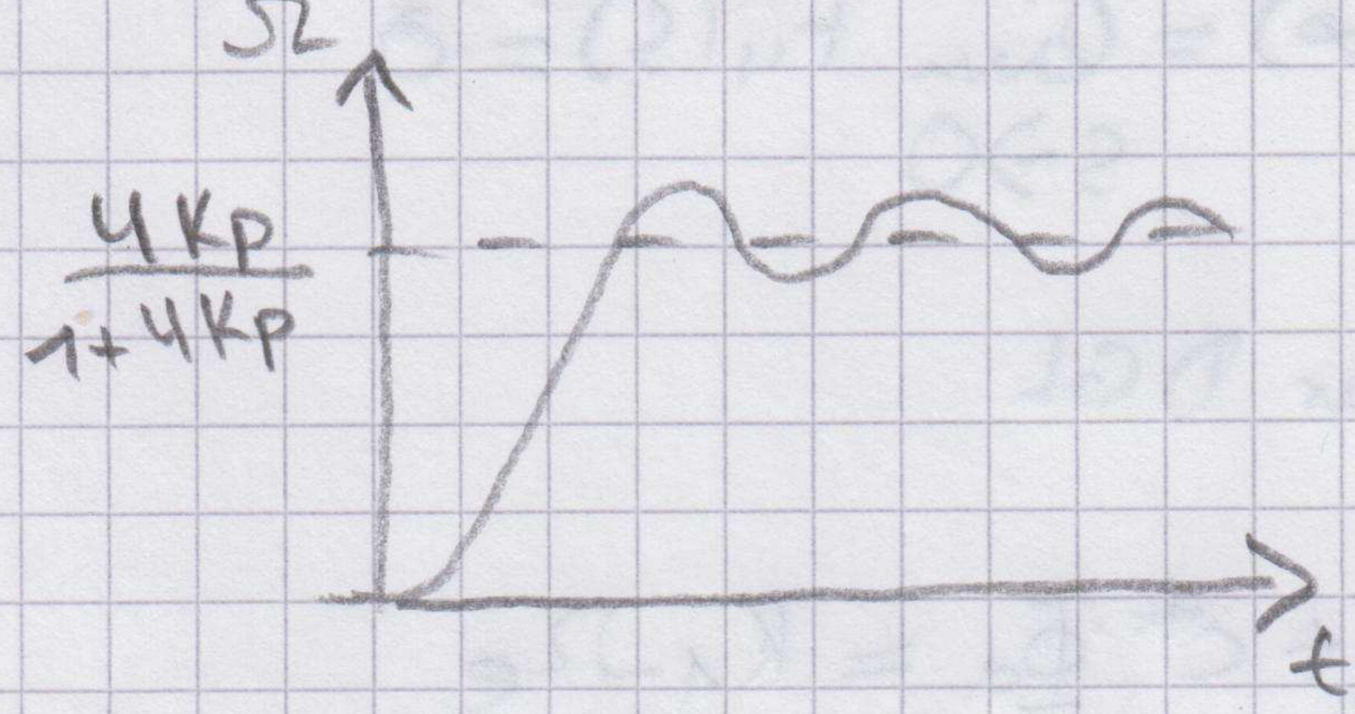
$= \frac{-4(16 s^2 + s)}{256 s^3 + 16 s^2 + (1 + K_P) \cdot s + 4K_I}$



(5) Führungsverhalten : $F_3(s) = \frac{\Omega}{U_W}$

$K_I = 0; K_P \neq 0$

$$\begin{cases} \Omega(t=0^+) = \lim_{s\to\infty} F_3(s) = 0 \\ \Omega(t\to\infty) = \lim_{s\to 0} F_3(s) = \frac{4K_P}{1+4K_P} \end{cases}$$



$K_I \neq 0; K_P = 0$

$$\begin{cases} \Omega(t=0^+) = \lim_{s\to\infty} F_3(s) = 0 \\ \Omega(t\to\infty) = \lim_{s\to 0} F_3(s) = 1 \end{cases}$$

$K_I \neq 0; K_P \neq 0$

$$\begin{cases} \Omega(t=0^+) = \lim_{s\to\infty} F_3(s) = 0 \\ \Omega(t\to\infty) = \lim_{s\to 0} F_3(s) = 1 \end{cases}$$

Störverhalten : $F_4(s) = \frac{\Omega}{M_b}$

$K_I = 0; K_P \neq 0$

$$\begin{cases} \Omega(t=0^+) = \lim_{s\to\infty} F_4(s) = 0 \\ \Omega(t\to\infty) = \lim_{s\to 0} F_4(s) = \frac{-4}{1+4K_P} \end{cases}$$

$K_I \neq 0 ; K_P = 0 \begin{cases} \Omega(t=0^+) = \lim\limits_{s\to\infty} F_4(s) = 0 \\ \Omega(t\to\infty) = \lim\limits_{s\to 0} F_4(s) = 0 \end{cases}$

$K_I \neq 0 ; K_P \neq 0 \begin{cases} \Omega(t=0^+) = \lim\limits_{s\to\infty} F_4(s) = 0 \\ \Omega(t\to\infty) = \lim\limits_{s\to 0} F_4(s) = 0 \end{cases}$

11.4.12 2) ① Transformation der DGL

$$J \cdot s^2 \cdot \Phi_a + d^* \cdot s \cdot \Phi_a + c^* \Phi_a = K_1 \Omega_e$$

$$\Rightarrow F(s) = \frac{\Phi_a}{\Omega_e} = \frac{K_1}{Js^2 + d^* s + c^*} = \frac{\frac{U_a}{K_2}}{\Omega_e}$$

$$F_1(s) = \frac{U_a}{\Omega_e} = \frac{K_1 \cdot K_2}{Js^2 + d^* s + c^*} \quad (PT_2)$$

- Produkt
- Verzögern

} 2. Ordn.

$$F_2(s) = \frac{U_a}{\Phi_e} = \frac{K_1 \cdot K_2 \cdot s}{J \cdot s^2 + d^* s + c^*} \quad (DT2)$$

- Differenzieren
- Verzögern

} 2. Ordn.

② a) Anfangswert: $U_a(t=0^+) = \lim\limits_{s\to\infty} \left[s \cdot F_2(s) \cdot \Phi_e(s) \right]$

Endwert: $U_a(t\to\infty) = \lim\limits_{s\to 0} \left[s \cdot F_2(s) \cdot \Phi_e(s) \right]$

i) $\Phi_e(t) = \delta(t) \circ\!\!-\!\!\bullet \Phi_e(s) = 1$

$$\Rightarrow U_a(t=0^+) = \frac{K_1 \cdot K_2}{J}$$

$$\Rightarrow U_a(t\to\infty) = 0$$

ii) $\Phi_e(t) = \sigma(t) \circ\!\!-\!\!\bullet \Phi_e(s) = \frac{1}{s}$

$$\Rightarrow U_a(t=0^+) = 0$$

$$\Rightarrow U_a(t\to\infty) = 0$$

iii) $\Phi_e(t) = t \cdot \sigma(t) \circ\!\!-\!\!\bullet \Phi_e(s) = \frac{1}{s^2}$

$\Rightarrow U_a(t = 0^+) = 0$

$\Rightarrow U_a(t \to \infty) = \frac{K_1 \cdot K_2}{C^*}$

b) $F_1(s) = \frac{K \cdot \omega_0^2}{s^2 + 2 \cdot D \cdot \omega_0 \cdot s + \omega_0^2} = \frac{\frac{K_1 \cdot K_2}{J}}{s^2 + \frac{d^*}{J} \cdot s + \frac{C^*}{J}}$

$= \frac{\frac{K_1 \cdot K_2}{C^*} \cdot \frac{C^*}{J}}{s^2 + \frac{d^*}{J} \cdot s + \frac{C^*}{J}}$

$\Rightarrow \ddot{U} = 25\% = D \approx 0{,}4$

$\Rightarrow T_e \approx 3{,}4s \Rightarrow \omega_e = \frac{2\pi}{T_e} = 1{,}85$

- Endwert: $\frac{K_1 \cdot K_2}{C^*} = 20 \Rightarrow C^* = 2$
- $\frac{C^*}{J} = \omega_0^2 = \left(\frac{\omega_e}{\sqrt{1-D^2}}\right)^2 \Rightarrow J = 0{,}5$
- $2 \cdot D \cdot \omega_0 = \frac{d^*}{J} \Rightarrow d^* = 2 \cdot D \cdot \omega_0 \cdot J = 0{,}8$

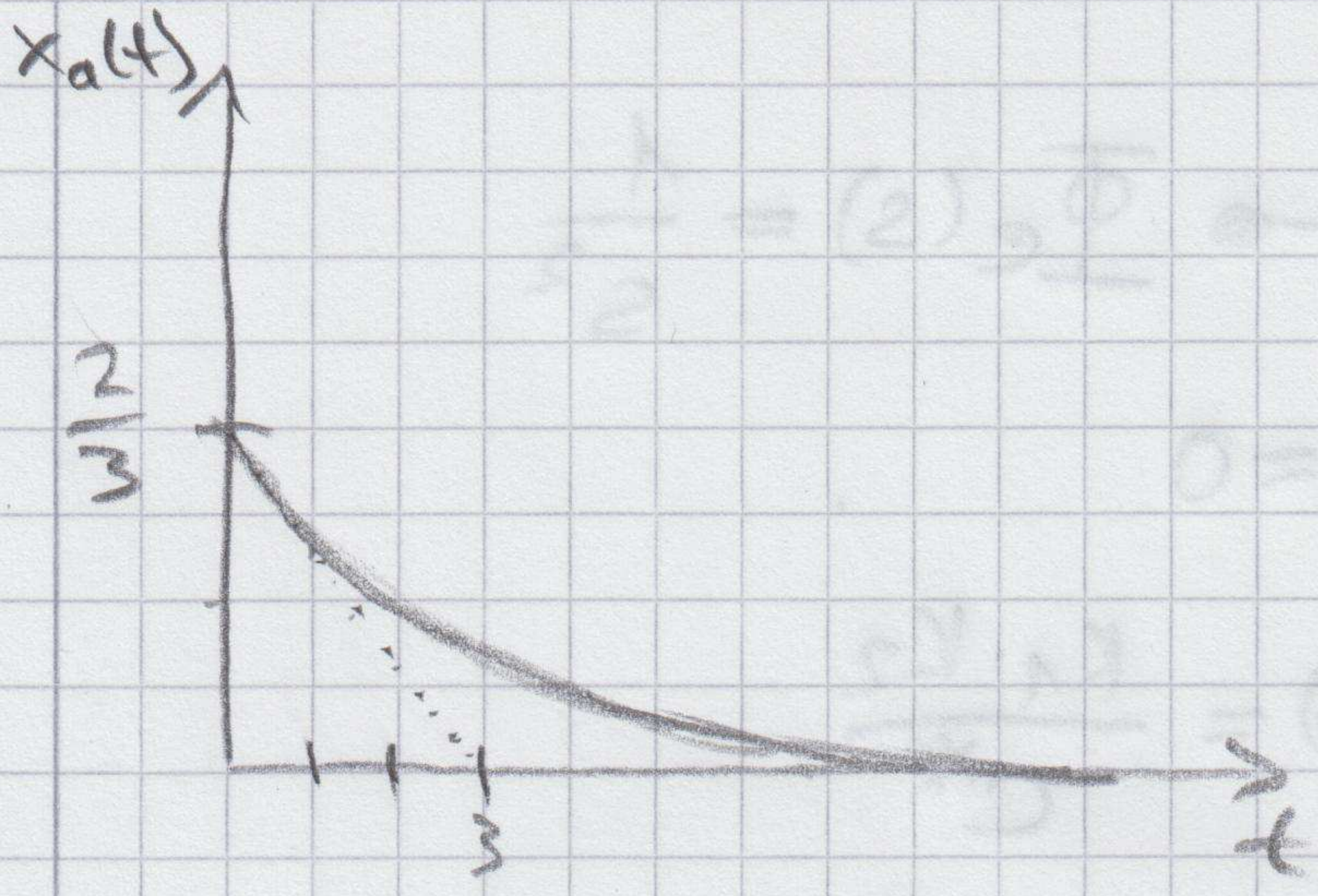
3) ① $F(s) = \frac{2}{1+3s} = \frac{X_a(s)}{X_e(s)}$

$X_a(s) = F(s) \cdot X_e(s)$

$x_a(t) = \mathcal{L}^{-1}\left\{\frac{2}{1+3s} \cdot X_e(s)\right\}$

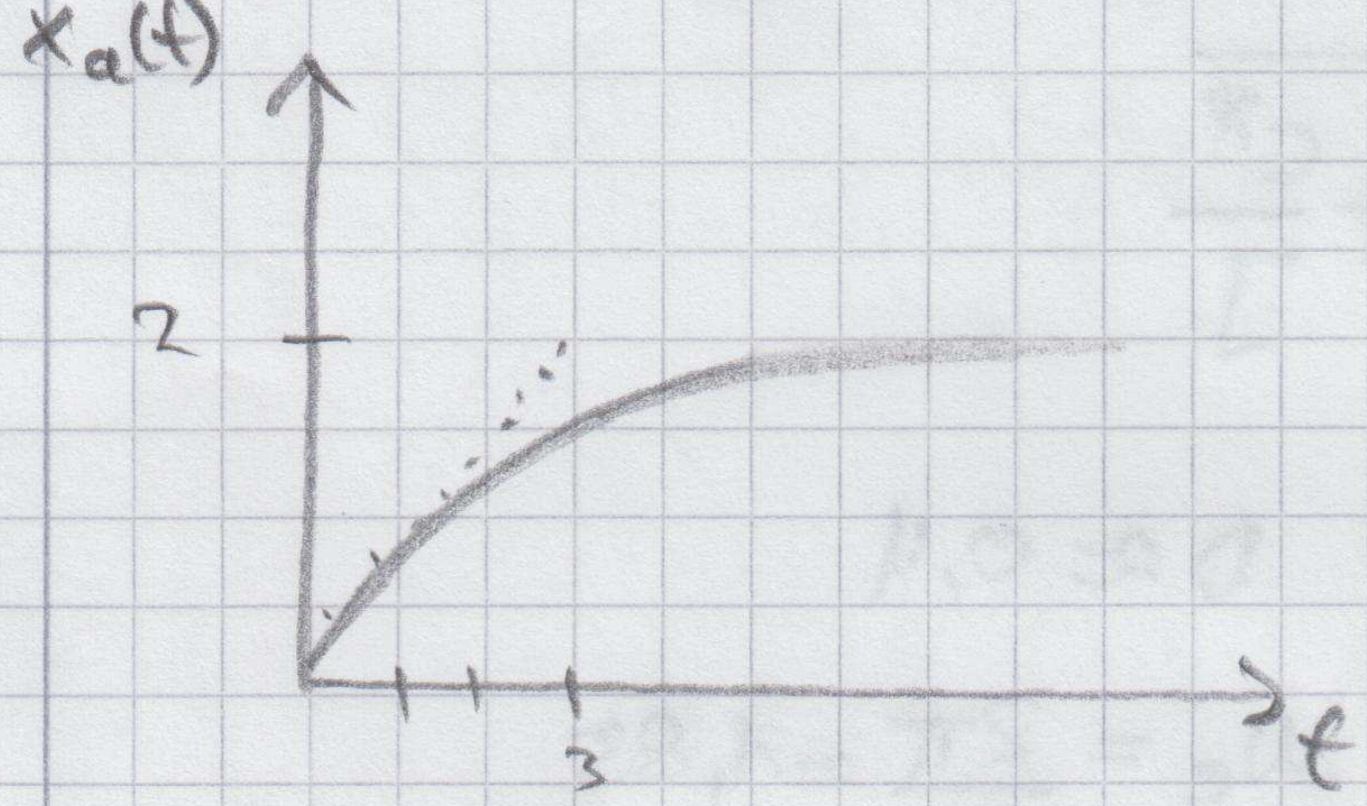
a) $X_e(s) = 1$ (Impulse)

$x_a(t) = \mathcal{L}^{-1}\left\{\frac{2}{1+3s}\right\} = \frac{2}{3} \cdot \mathcal{L}^{-1}\left\{\frac{3}{1+3s}\right\} = \frac{2}{3} \cdot e^{-\frac{t}{3}} \cdot \sigma(t)$



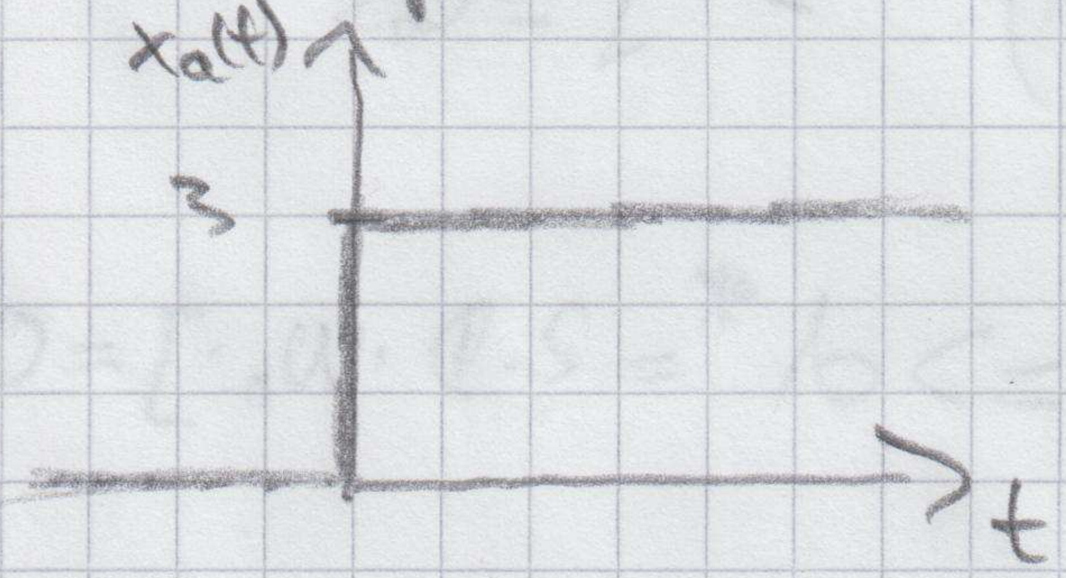
b) $x_e(s) = \frac{1}{s}$ (Sprung)

$$x_a(t) = L^{-1}\left\{\frac{2}{1+3s}\cdot\frac{1}{s}\right\} = 2\cdot\left(1-e^{-\frac{t}{3}}\right)\cdot\sigma(t)$$

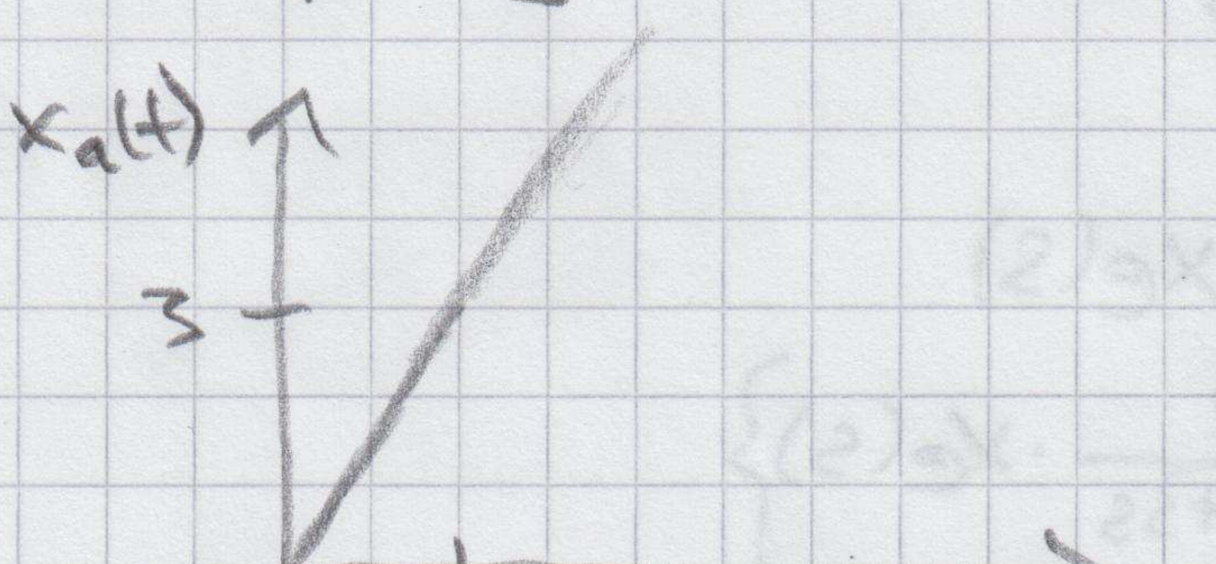


② $F(s) = \frac{3}{s}$

a) Impuls: $x_a(t) = L^{-1}\left\{\frac{3}{s}\right\} = 3\cdot\sigma(t)$



b) Sprung: $x_a(t) = L^{-1}\left\{\frac{3}{s}\cdot\frac{1}{s}\right\} = 3\cdot t\cdot\sigma(t)$



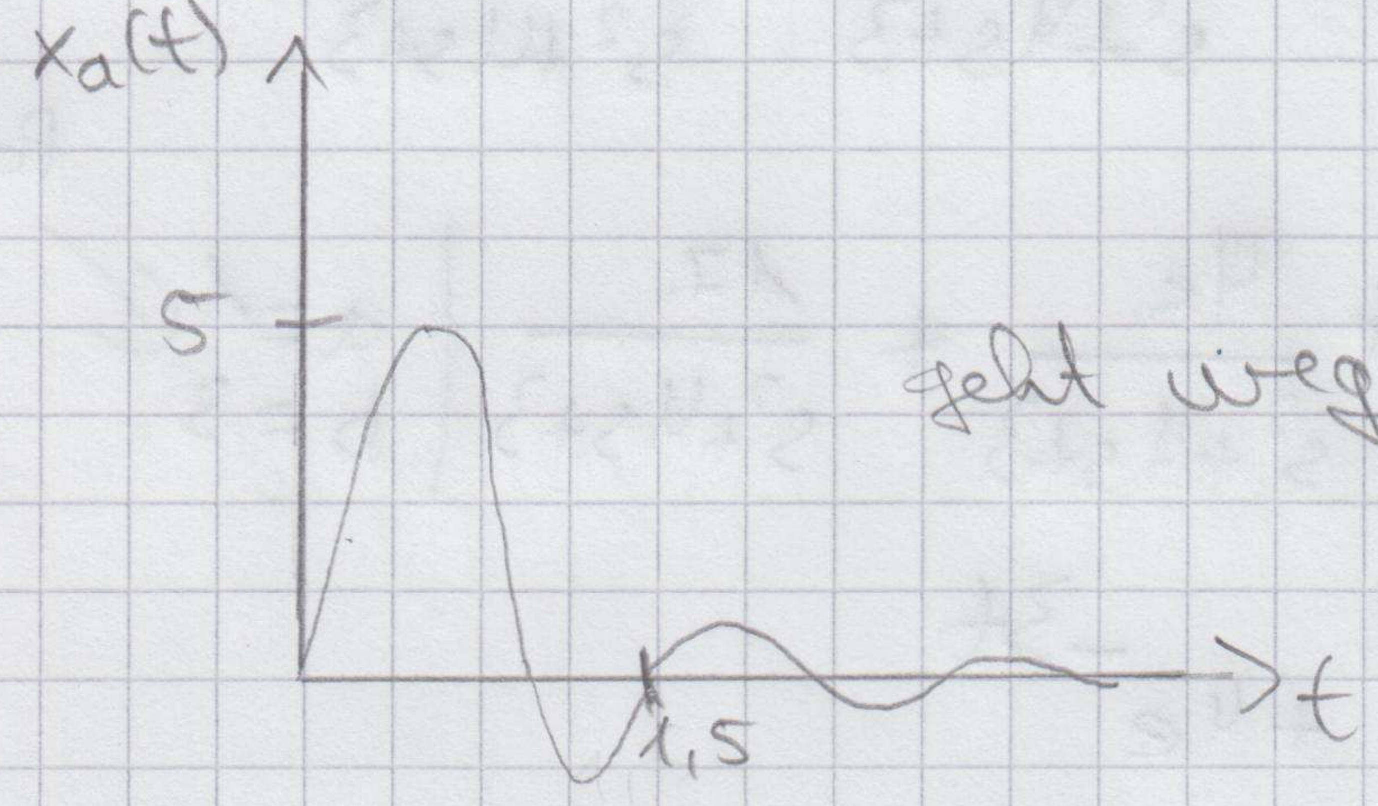
③ $F(s) = \frac{50}{s^2+6s+25} = \frac{K\cdot\omega_0^2}{s^2+2\cdot D\cdot\omega_0\cdot s+\omega_0^2}$

$\Rightarrow \omega_0^2 = 25 \Rightarrow \omega_0 = 5$

$\Rightarrow 2\cdot D\cdot\omega_0 = 6 \Rightarrow D = \frac{6}{2\cdot\omega_0} = 0{,}6$

$\Rightarrow \omega_e = \omega_0\sqrt{1-D^2} = 4$

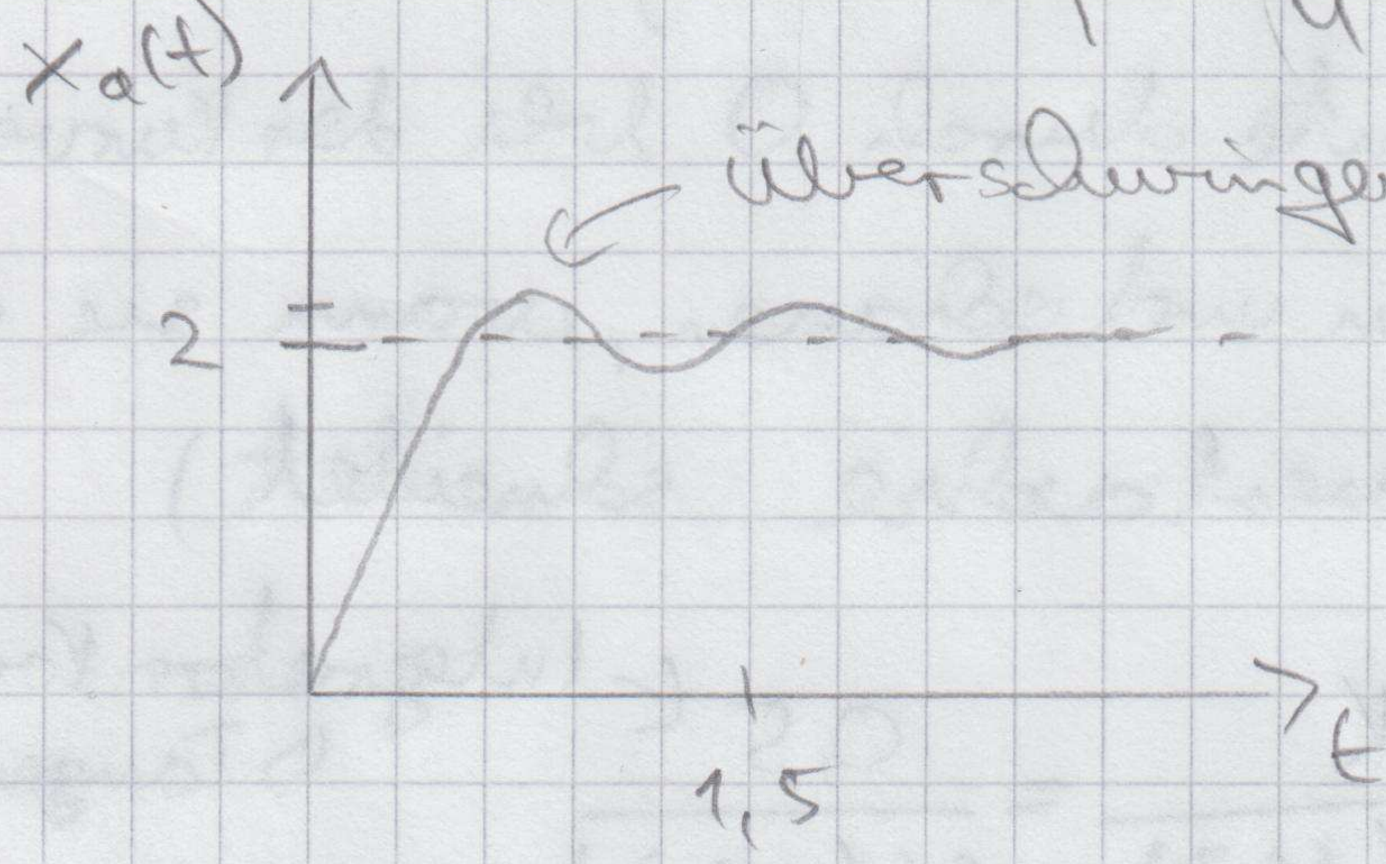
a) Impuls: $x_a(t) = \mathcal{L}^{-1}\left\{\frac{50}{s^2+6s+25}\right\} = 50\cdot\frac{1}{4}e^{-3t}\cdot\sin(4t)\cdot\sigma(t)$



geht wegen dem e^{-t} $\to\infty$ gegen 0?

b) Sprung: $x_a(t) = \mathcal{L}^{-1}\left\{\frac{50}{s^2+6s+25}\cdot\frac{1}{s}\right\}$

$= 2\cdot\left(1-\frac{e^{-3t}}{4}\cdot\left(3\cdot\sin(4t)+4\cdot\cos(4t)\right)\right)\cdot\sigma(t)$



4) ①

$\mathcal{L}\{\dot{x}_a + x_a = 5\cdot x_e\}$

$\Rightarrow s\cdot x_a(s) - x_a(t=0) + x_a(s) = 5\cdot x_e(s)$

$\Rightarrow x_a(s) = \frac{x_a(t=0)}{1+s} + \frac{5\cdot x_e(t)}{1+s}$

$\Rightarrow x_e(s) = \frac{2}{s}$

$$x_a(t) = \mathcal{L}^{-1}\left\{\frac{2}{1+s} + \frac{10}{s(s+1)}\right\} = 2e^{-t} + 10(1-e^{-t})$$
$$= 10 - 8e^{-t}$$

(2)

$$\mathcal{L}\{\ddot{x}_a(t) + 4\dot{x}_a(t) + 3x_a(t) = 13x_e(t)\}$$

$$\Rightarrow s^2 x_a(t) - s \cdot x_a(t=0) - \dot{x}_a(t=0) + 4 \cdot s \cdot x_a(s) - 4x_a(t=0) + 3x_a(s) = 13x_e(s)$$

$$\Rightarrow x_a(s)(s^2+4s+3) = 13x_e(s) + s \cdot 4 + 1 + 16$$

$$\Rightarrow x_a(s) = \frac{13x_e(s)}{s^2+4s+3} + \frac{4s}{s^2+4s+3} + \frac{17}{s^2+4s+3} =$$

Polstellen

$$= \frac{39}{s(s^2+4s+3)} + \frac{4s}{s^2+4s+3} + \frac{17}{s^2+4s+3} \quad \Big| \; a=1, \; b=3$$

$$\Rightarrow x_a(t) = 13 - 13e^{-t} + 4e^{-3t}$$

5) (1) a) PT_1 b) IT_1 c) PT_2

(2) a) $F(s) = \frac{K}{1+sT} = \frac{3}{1+s \cdot 2,5}$

(Tangente durch 0 bei der Kurve anlegen und schauen, wann sie die Endwertachse schneidet)

b) $F(s) = \frac{K}{s(1+sT)} = \frac{0,5}{s(1+s \cdot 2)}$ ← Integrator Konst.
↳ Tangente anlegen

c) $F(s) = \frac{K\omega_0^2}{s^2 + 2 \cdot D\omega_0 s + \omega_0^2}$

Endwert: 2 $\Rightarrow K = 2$

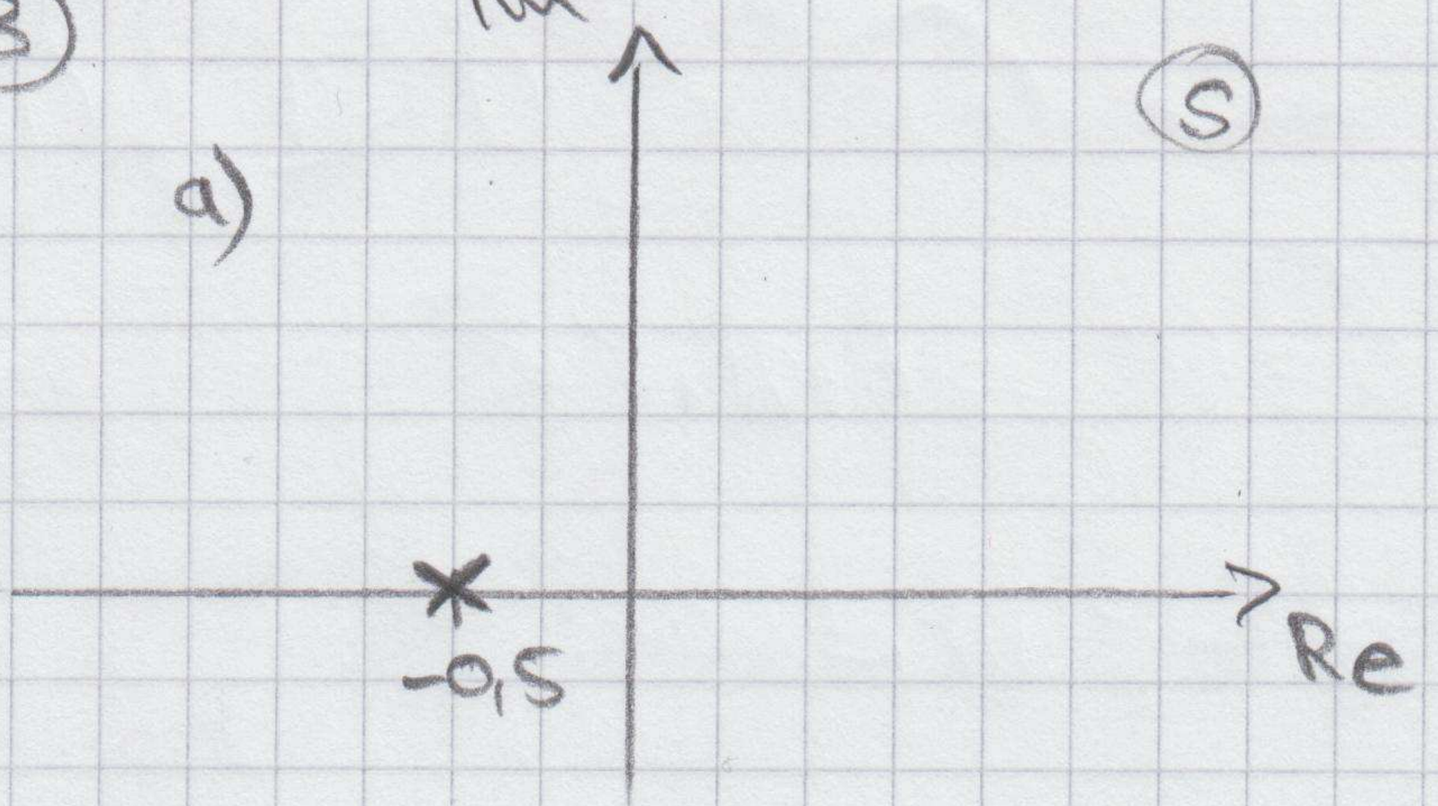
$\ddot{U} \approx 85\% \Rightarrow D = 0{,}05$ (← Grafik)

$T_e \approx 3{,}1 \Rightarrow \omega_e = \frac{2\pi}{T_e} \approx 2$

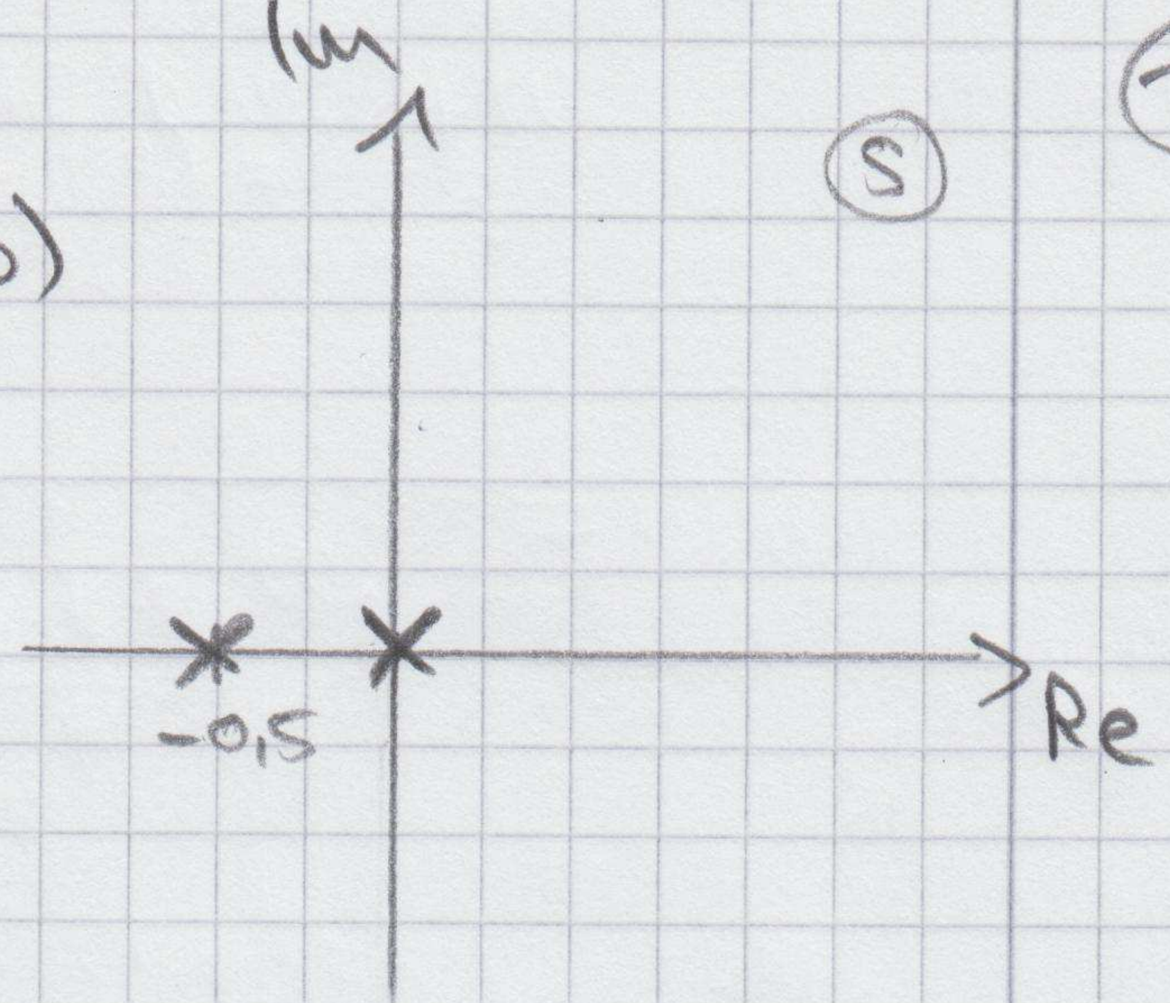
$= \omega_0 = \frac{\omega_e}{\sqrt{1-D^2}} \approx 2$

③ ⑰

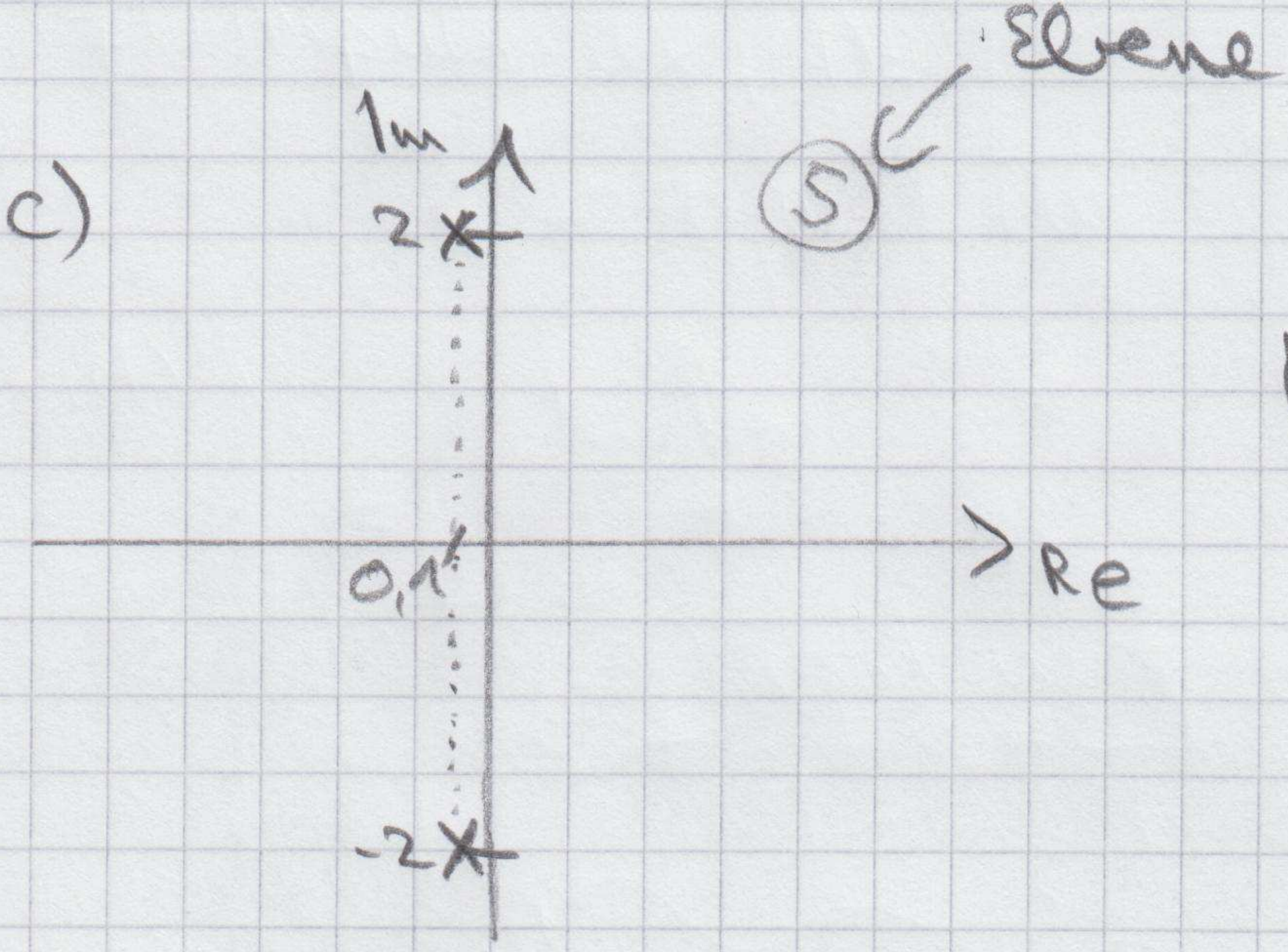
a)



b)



c)



$p_{1,2} = -0{,}1 \pm j \cdot 2$

6) 1. $F(s) = \frac{K \cdot s}{1 + T \cdot s} = \frac{0{,}5 \cdot s}{1 + 0{,}5 \cdot s}$

$x(t=0) = \lim_{s \to \infty} F(s)$

$x(t \to \infty) = \lim_{s \to 0} F(s)$

↳ c)

2. $F(s) = \frac{K}{(s+1)(s+2)} = \frac{1}{(s+1)(s+2)}$

↳ e)

3. → b)

$F(s) = \frac{K\omega_0^2}{s^2 + 2D\omega_0 s + \omega_0^2}$

4. → a)

$F(s) = \frac{K(1 - T_1 \cdot s)}{(1 + T_2 \cdot s)}$

5. → d)

$F(s) = \frac{K_I}{s} = \frac{1}{s}$