30.4.2012 Übungsblatt 5

1) ① $F_1(s) = \frac{2}{s} \rightarrow F_1(j\omega) = \frac{2}{j\omega} = -j\frac{2}{\omega} = \frac{2}{\omega} \cdot e^{-j\frac{\pi}{2}}$

$\omega \rightarrow 0$:

$|F_1(j\omega)|_{\omega \rightarrow 0} = \infty$

$\measuredangle F_1(j\omega)|_{\omega \rightarrow 0} = \arg F_1(j\omega) = \arctan\left(\frac{\operatorname{Im} F_1(j\omega)}{\operatorname{Re} F_1(j\omega)}\right)\Big|_{\omega \rightarrow 0}$

$= -\frac{\pi}{2} = -90°$

$\omega \rightarrow \infty$

$|F_1(j\omega)|_{\omega \rightarrow \infty} = 0$

$\measuredangle F_1(j\omega)|_{\omega \rightarrow \infty} = \arg F_1(j\omega) = \arctan\left(\frac{\operatorname{Im} F_1(j\omega)}{\operatorname{Re} F_1(j\omega)}\right)\Big|_{\omega \rightarrow \infty}$

$= -\frac{\pi}{2} = -90°$

$F_2(s) = \frac{2}{1+3s} \rightarrow F_2(j\omega) = \frac{2}{1+3j\omega} = \frac{2(1-3j\omega)}{1+9\omega^2}$

$= \frac{2-6j\omega}{1+9\omega^2}$

$\omega \rightarrow 0$:

$|F_2(j\omega)|_{\omega \rightarrow 0} = 2$

$\measuredangle F_2(j\omega)|_{\omega \rightarrow 0} = \arctan\left(\frac{\operatorname{Im} F_2(j\omega)}{\operatorname{Re} F_2(j\omega)}\right) =$

$= \arctan\left(\frac{-6\omega}{2}\right)\Big|_{\omega \rightarrow 0} = 0°$

$\omega \rightarrow \infty$

$|F_2(j\omega)|_{\omega \rightarrow \infty} = 0$

$\measuredangle F_2(j\omega)|_{\omega \rightarrow \infty} = \arctan\left(\frac{-6\omega}{2}\right)\Big|_{\omega \rightarrow \infty} = -\frac{\pi}{2}$

<u>2.5.2012</u>

$$F_3(j\omega) = \frac{10}{(1+j\omega)(4+j\omega)} = \frac{10}{4+j\omega+4j\omega-\omega^2} = \frac{10}{(4-\omega^2)+j5\omega}$$

$$= \frac{10\left((4-\omega^2)-j\omega 5\right)}{(4-\omega^2)^2+25\omega^2}$$

$\underline{\omega \to 0}$: $|F_3(j\omega)| = 2{,}5$

$\sphericalangle F_3(j\omega) = 0°$

$\underline{\omega \to \infty}$: $|F_3(j\omega)| = 0$

$\sphericalangle F_3(j\omega) = -180°$

$$F_4(j\omega) = \frac{100}{(s+1)^2(s+4)} = \frac{100}{s^3+4s^2+2s^2+8s+s+4}$$

$$= \frac{100}{-j\omega^3-6\omega^2+9j\omega+4} = \frac{100}{(4-6\omega^2)+j(9\omega-\omega^3)}$$

$$= \frac{100\left((4-6\omega^2)-j(9\omega-\omega^3)\right)}{(4-6\omega^2)^2+(9\omega-\omega^3)^2}$$

$\underline{\omega \to 0}$: $|F_4(j\omega)| = 25$

$\sphericalangle F_4(j\omega) = 0°$

$\underline{\omega \to \infty}$: $|F_4(j\omega)| = 0$

$\sphericalangle F_4(j\omega) = -270°$

(NICHT +90°)
(würde hierbei <u>vor</u> der Anregung reagieren)

(Im ausgeschwungenen Zustand macht es aber kein Unterschied)

(2) $F_3(j\omega)$: SP mit Imag-Achse

$\Rightarrow 4-\omega^2 \overset{!}{=} 0 \to \omega = 2$

$\Rightarrow F_3(j \cdot 2) = -j$

$F_4(j\omega)$: SP mit Imag-Achse

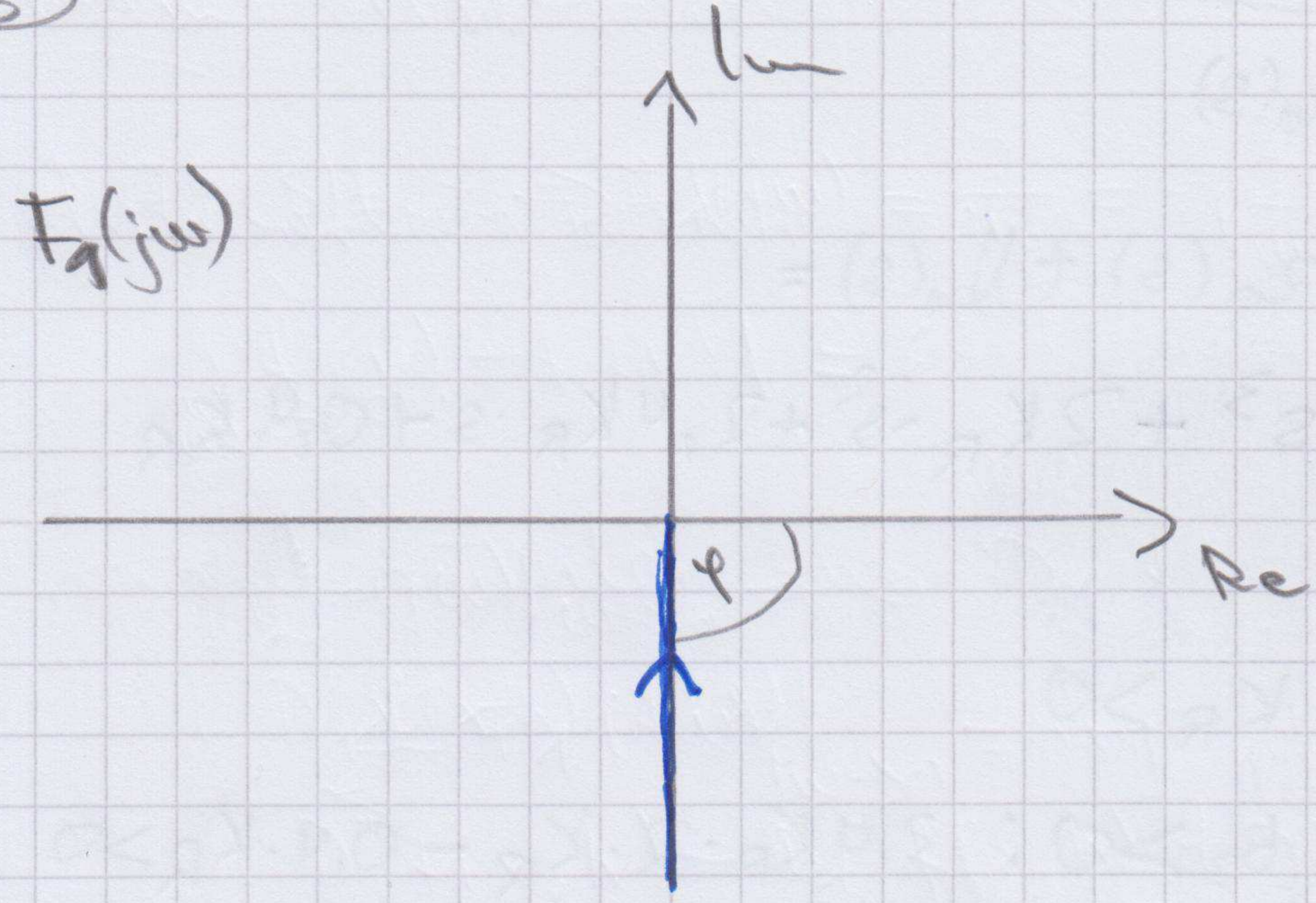
$\Rightarrow 4-6\omega^2 = 0 \to \omega = \sqrt{\frac{2}{3}}$

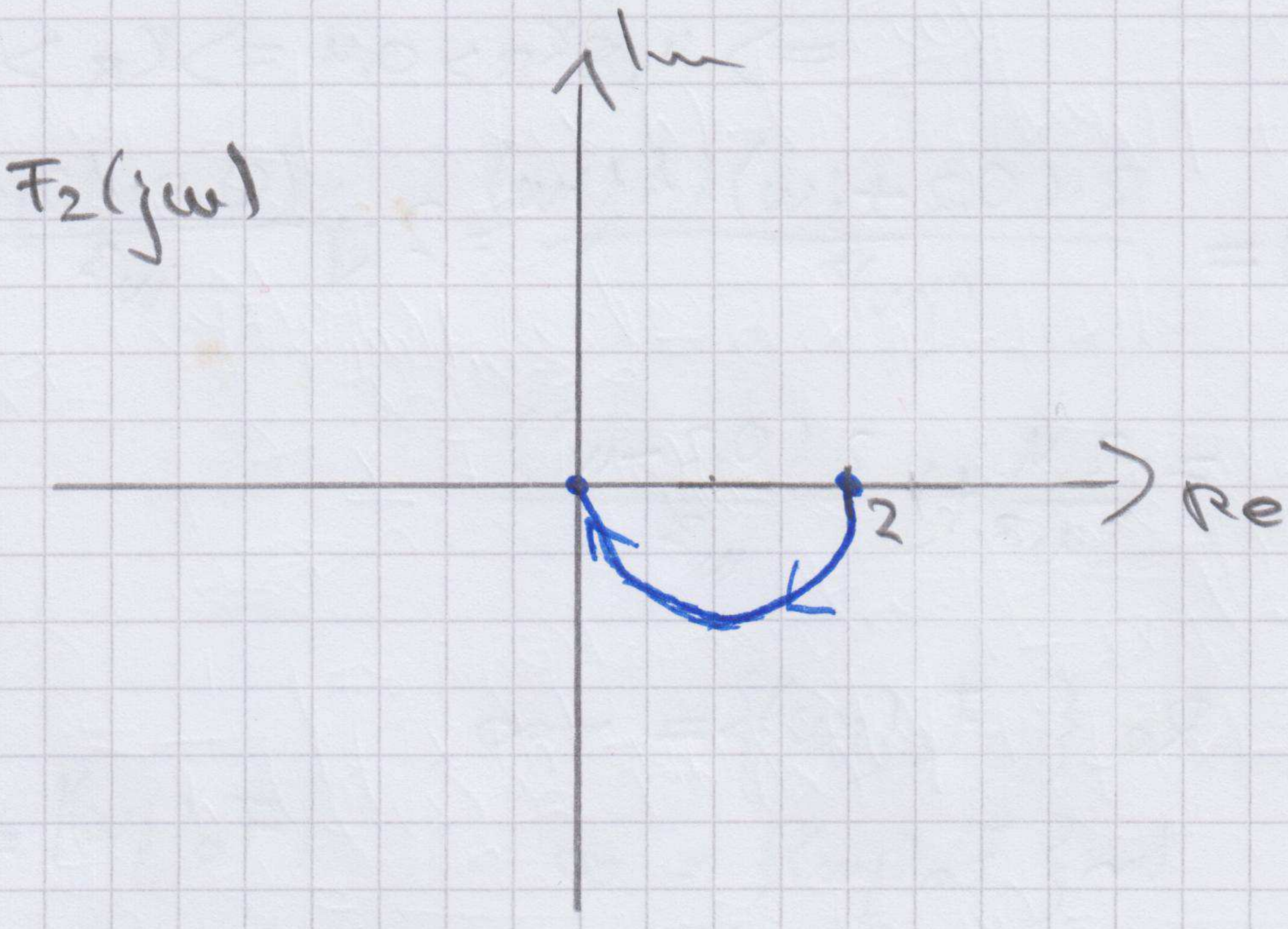
$\Rightarrow F_4\left(j\sqrt{\frac{2}{3}}\right) = -j14{,}7$

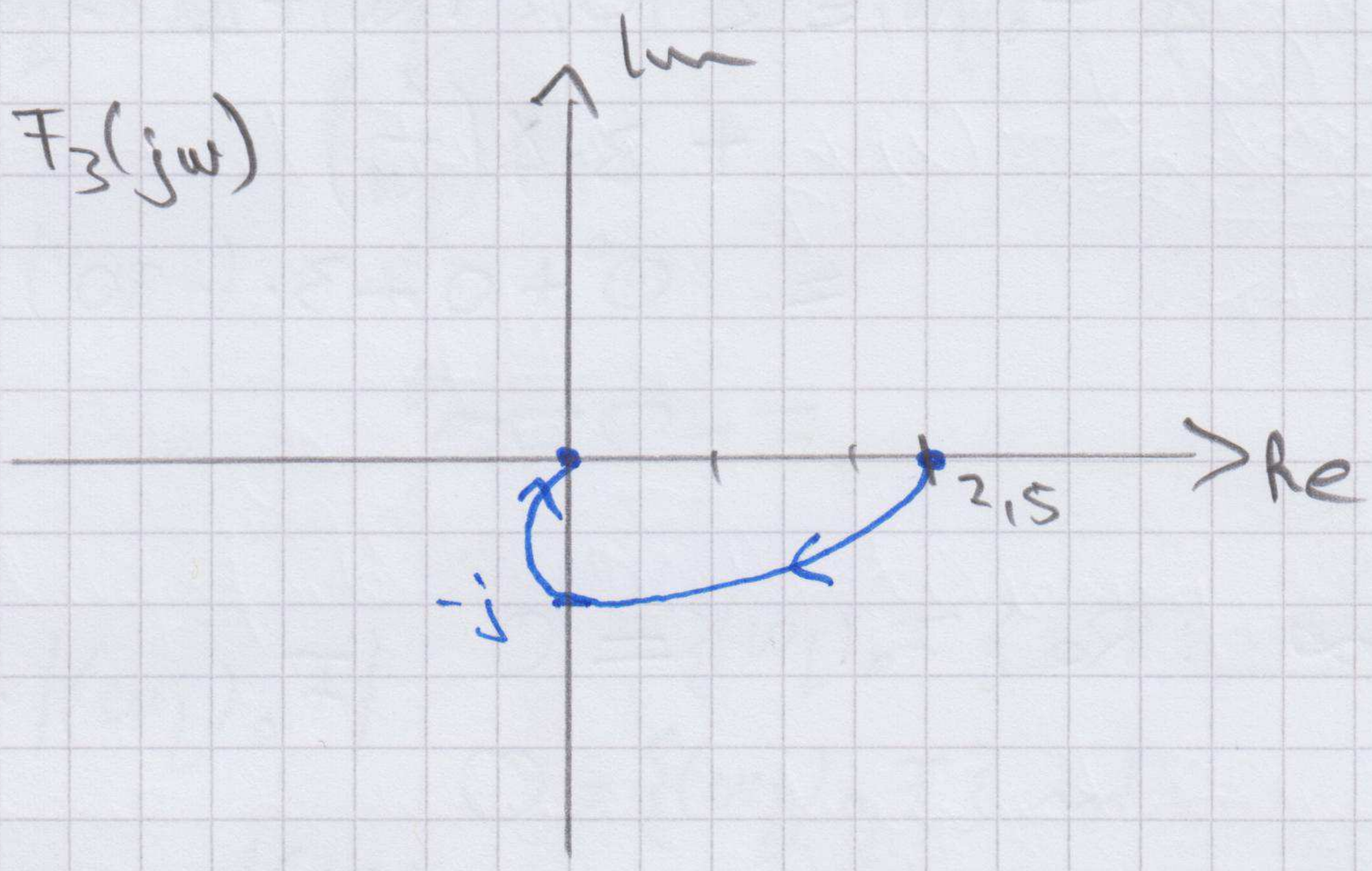
SP mit reeller-Achse:

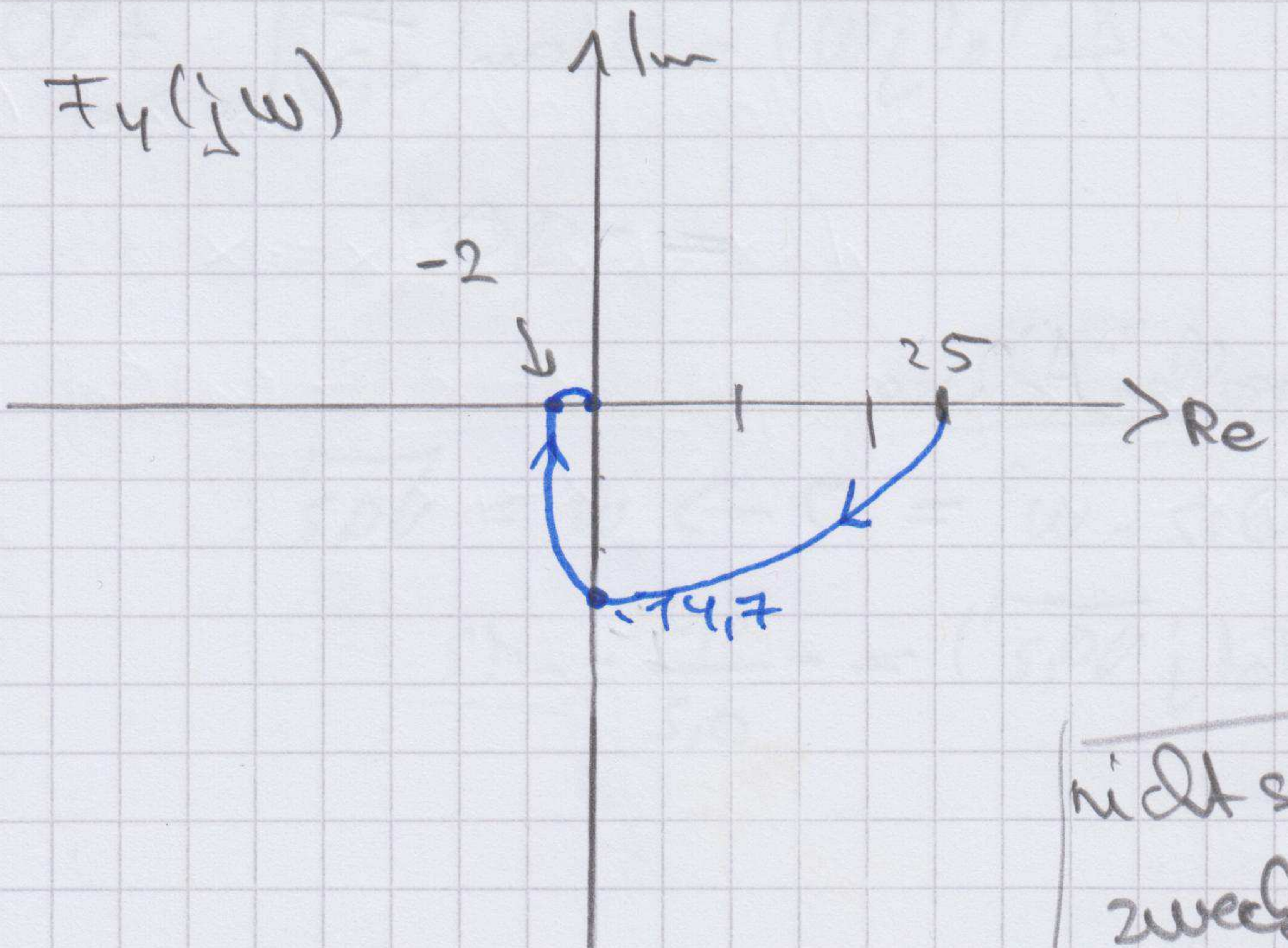
$\Rightarrow 9-\omega^2 = 0 \to \omega = 3$

$\Rightarrow F_4(j \cdot 3) = -2$









nicht schön, aber zweckmäßig !

(2) 1) $F_0(s) = \frac{Z_0(s)}{N_0(s)}$ (31)

$N_{RK}(s) = Z_0(s) + N_0(s) =$

$= s^3 + 2K_R \cdot s^2 + 2{,}4K_R \cdot s + 0{,}4K_R$

<u>Hurwitz</u>

$b_0 > 0 : K_R > 0$

$b_1 \cdot b_2 - b_0 \cdot b_3 > 0 : 2{,}4K_R \cdot 2 \cdot K_R - 0{,}4 \cdot K_R > 0$

$\Rightarrow 4{,}8K_R > 0{,}4 \Rightarrow K_R > \frac{1}{12}$

2) $F_0(j\omega) = \frac{2 \cdot (0{,}2 + j\omega)(1 + j\omega)}{(j\omega)^3} = 2 \cdot j \cdot \frac{(0{,}2 + 1{,}2j\omega - \omega^2)}{\omega^3}$

$= \frac{-2{,}4}{\omega^2} + j\frac{2(0{,}2 - \omega^2)}{\omega^3}$

<u>$\omega \to 0$</u>: $Re\{F_0(j\omega)\} = -\infty$

$Im\{F_0(j\omega)\} = \infty$ $|F_0(j\omega)| = \infty$

$\measuredangle F_0(j\omega) = \measuredangle(0{,}2 + j\omega) + \measuredangle(1 + j\omega)$

$+ 3 \cdot \measuredangle\left(\frac{1}{j\omega}\right)$

$= 0° + 0° + 3 \cdot (-90°)$

$= -270°$

<u>$\omega \to \infty$</u> : $Re\{F_0(j\omega)\} = 0$ $|F_0(j\omega)| = 0$

$Im\{F_0(j\omega)\} = 0$

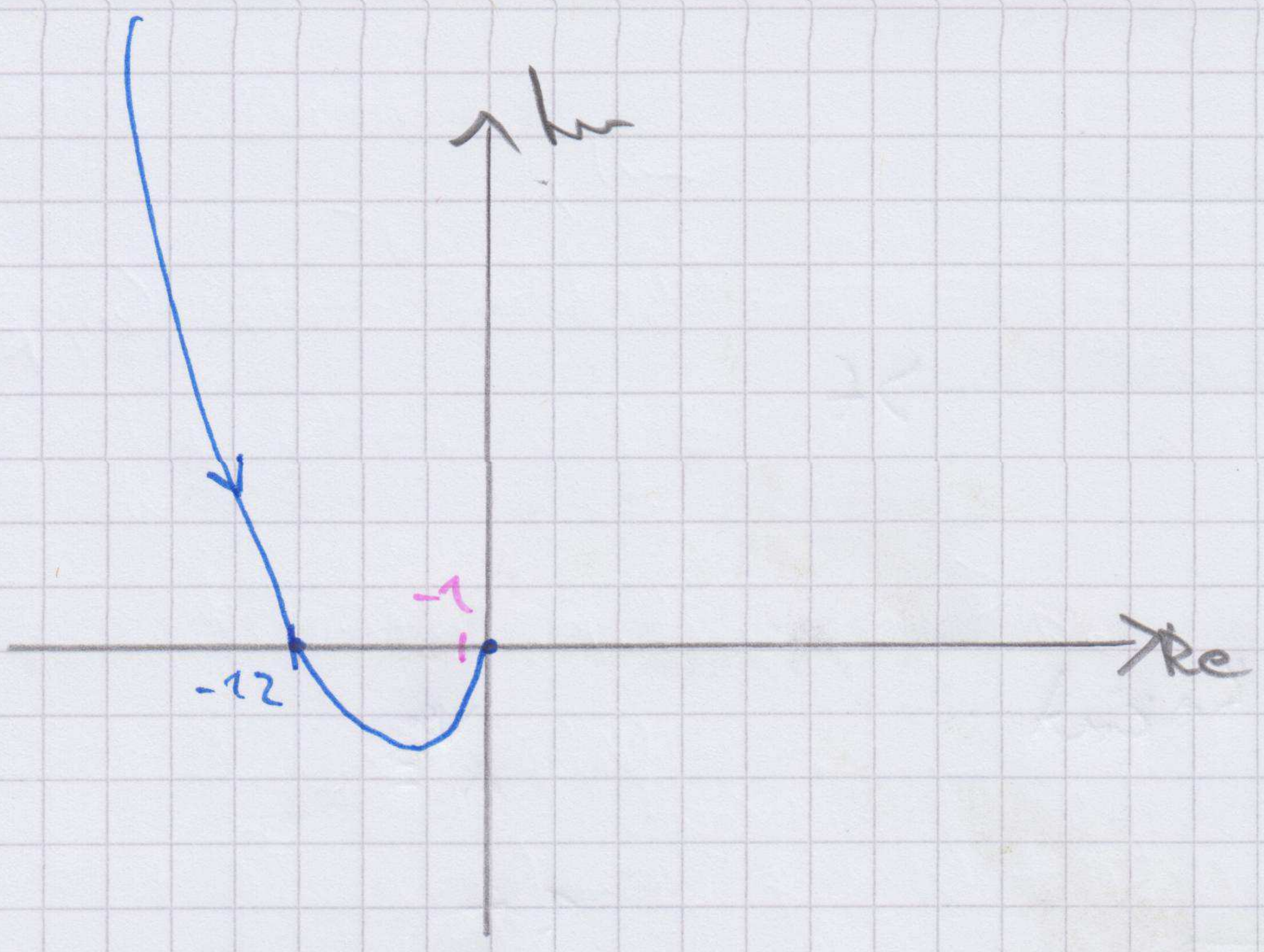
$\measuredangle F_0(j\omega) = atan\left(\frac{\omega}{0{,}2}\right)\Big|_{\omega \to \infty} + 90° - 270°$

$= -90°$

<u>SP mit reeller Achse</u>

$\Rightarrow 0{,}2 - \omega^2 = 0 \to \omega = \sqrt{0{,}2}$

$F_0(j\sqrt{0{,}2}) = -\frac{2{,}4}{0{,}2} = \underline{-12}$



↙ Anzahl der Schnittpunkte auf der Achse, links vom der Img-Achse

③ $n_a = 3$; $n_r = 0$

$\omega_{soll} = n_a \cdot \frac{\pi}{2} + n_r \cdot \pi = 270°$

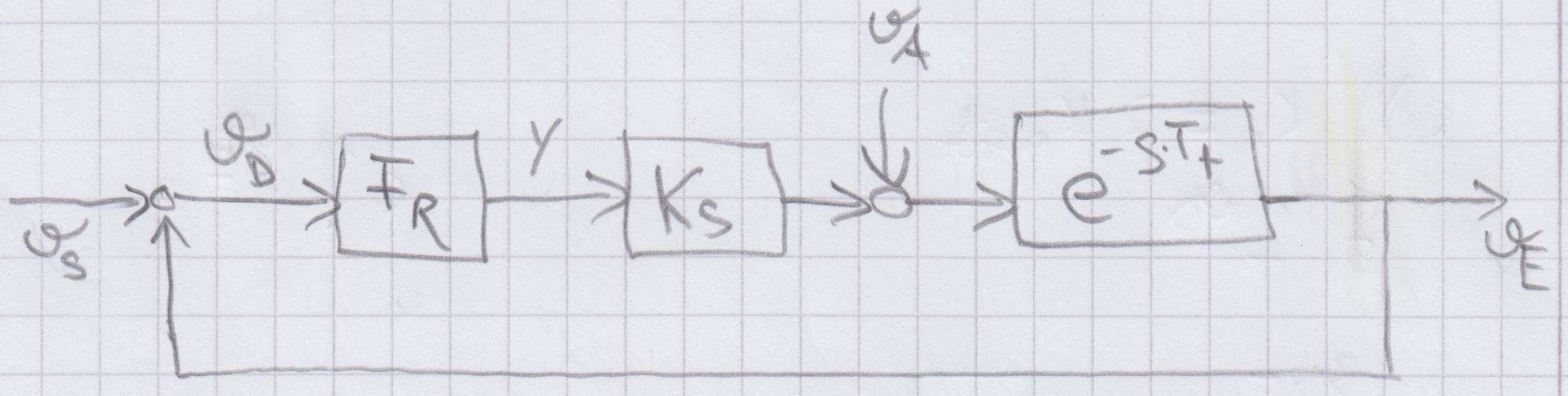
$\omega_{ist}(K_R = 1) = 270° \Rightarrow$ stabil $\Rightarrow K_R > \frac{1}{12}$

④ $F_0(j\omega) = -1$

$\Rightarrow Re\{F_0(j\omega)\} = -1 = -\frac{2{,}4 K_R}{\omega_{krit}^2} \Rightarrow K_{Rkrit} = \frac{1}{12}$

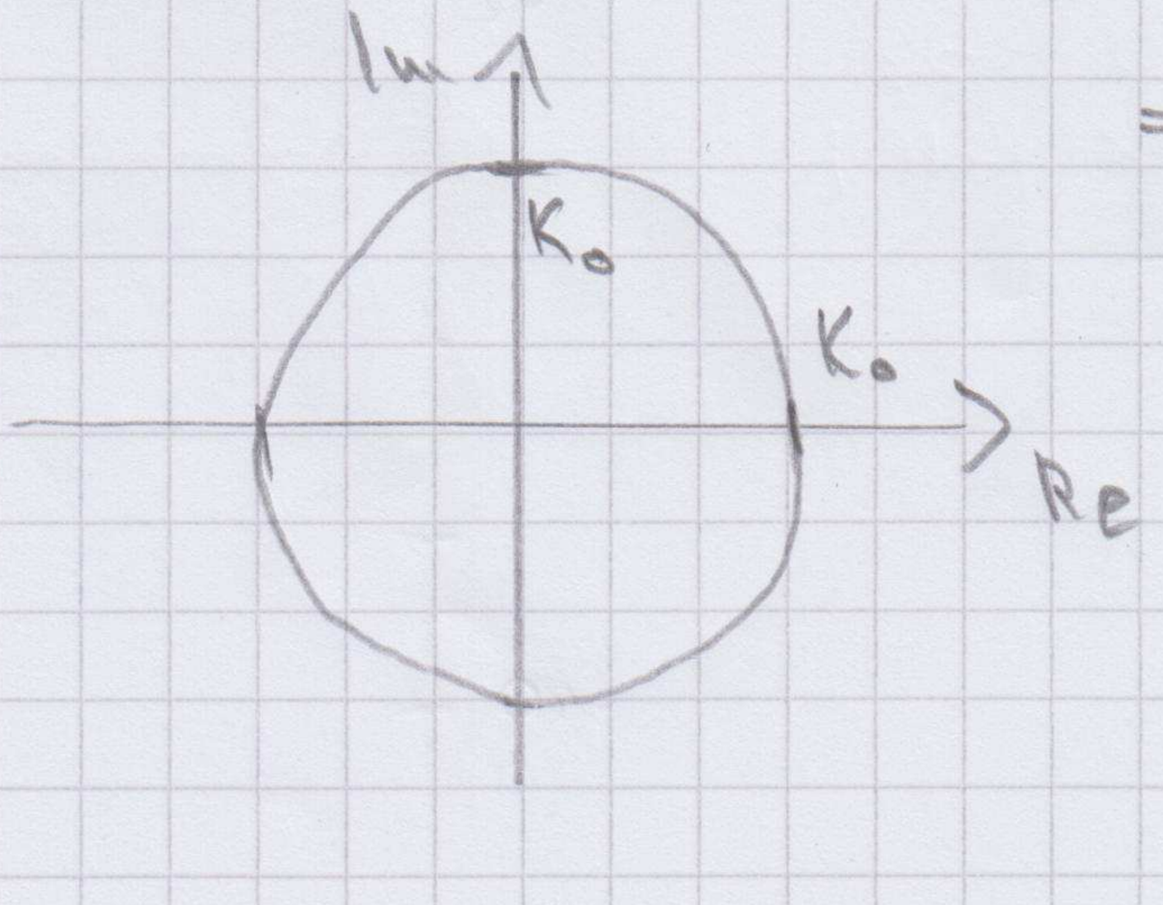
$\Rightarrow Im\{F_0(j\omega)\} = 0 \Rightarrow \omega = \sqrt{0{,}2} = \omega_{krit}$

3) ①



$T_t = \frac{V}{L} = 0{,}5h$

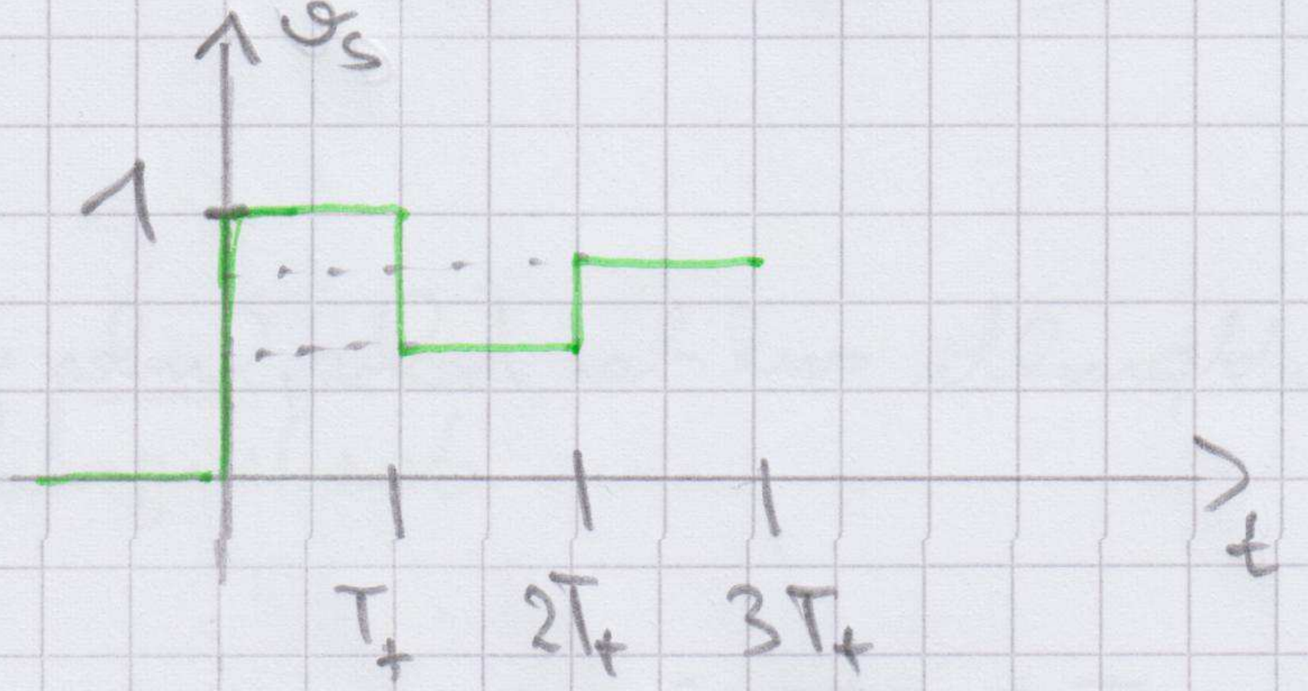
② $F_0(j\omega) = K_R \cdot K_S \cdot e^{-j\omega T_t} = K_0 \cdot e^{-j\omega T_t}$

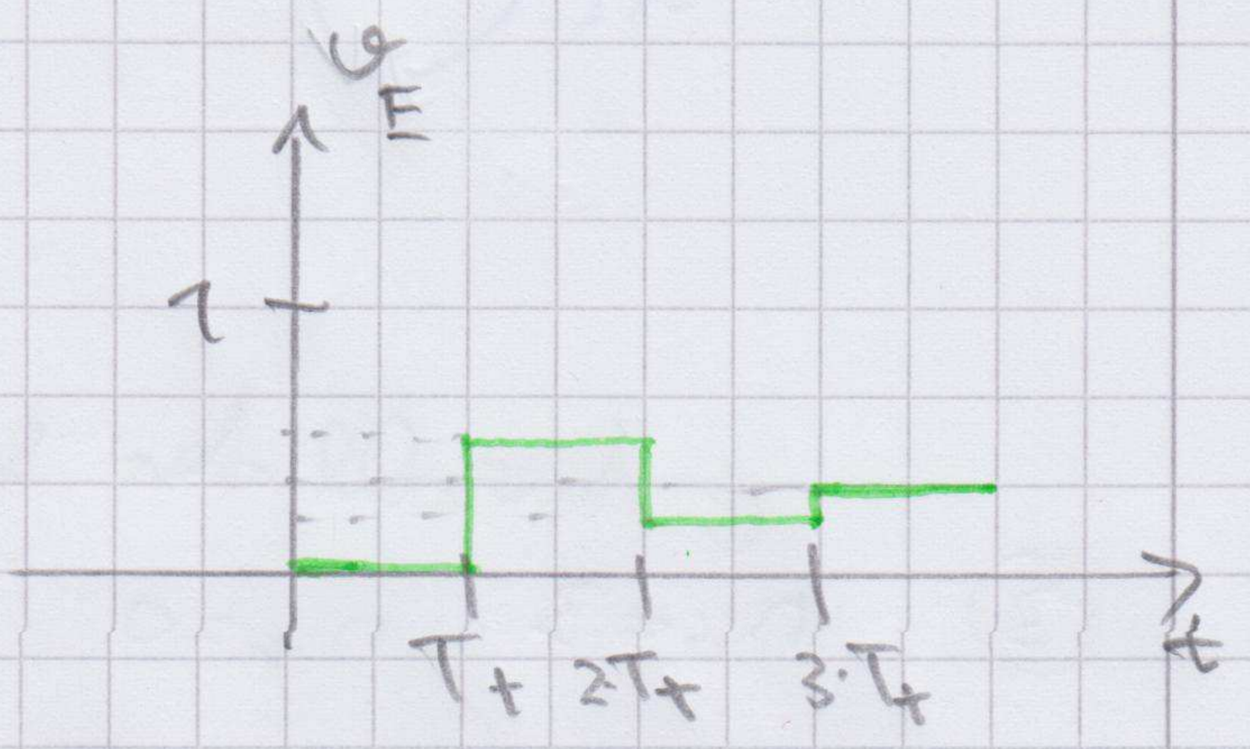


$\Rightarrow$ Linke-Hand-Regel

↳ Stabil $K_0 < 1$

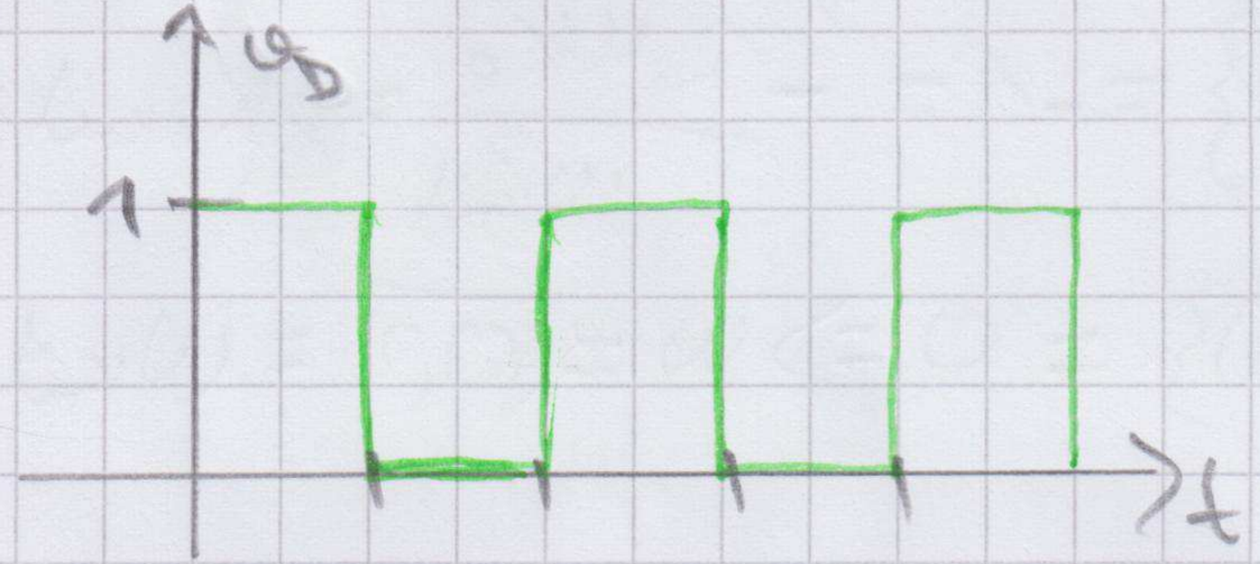
a) $K_0 = 0{,}5 \cdot K_{0krit}$

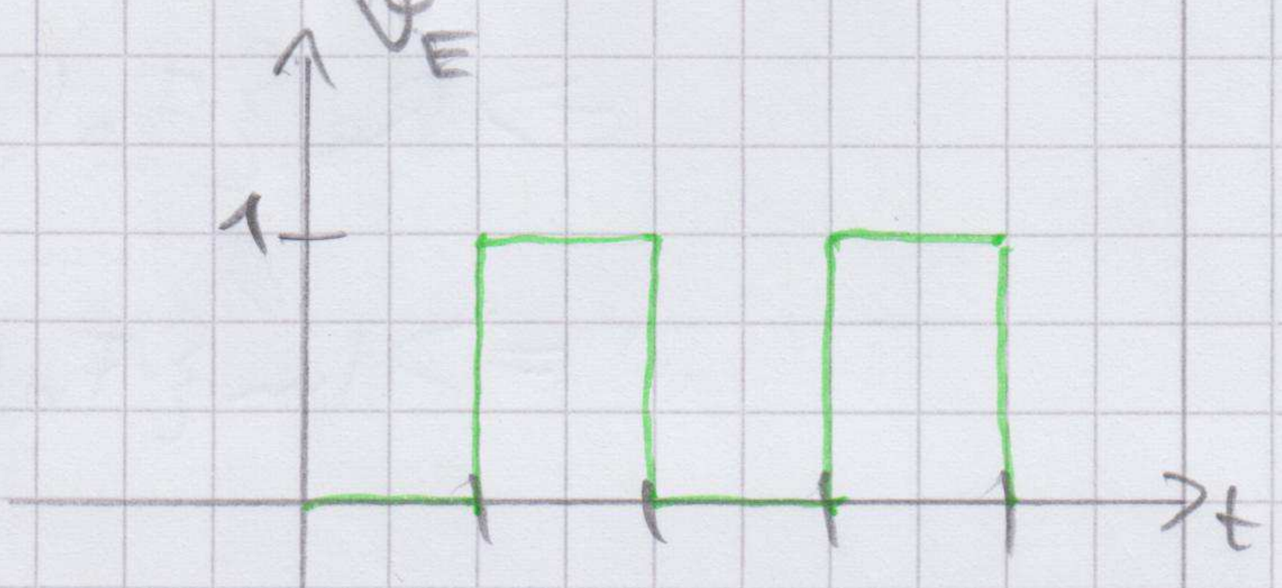




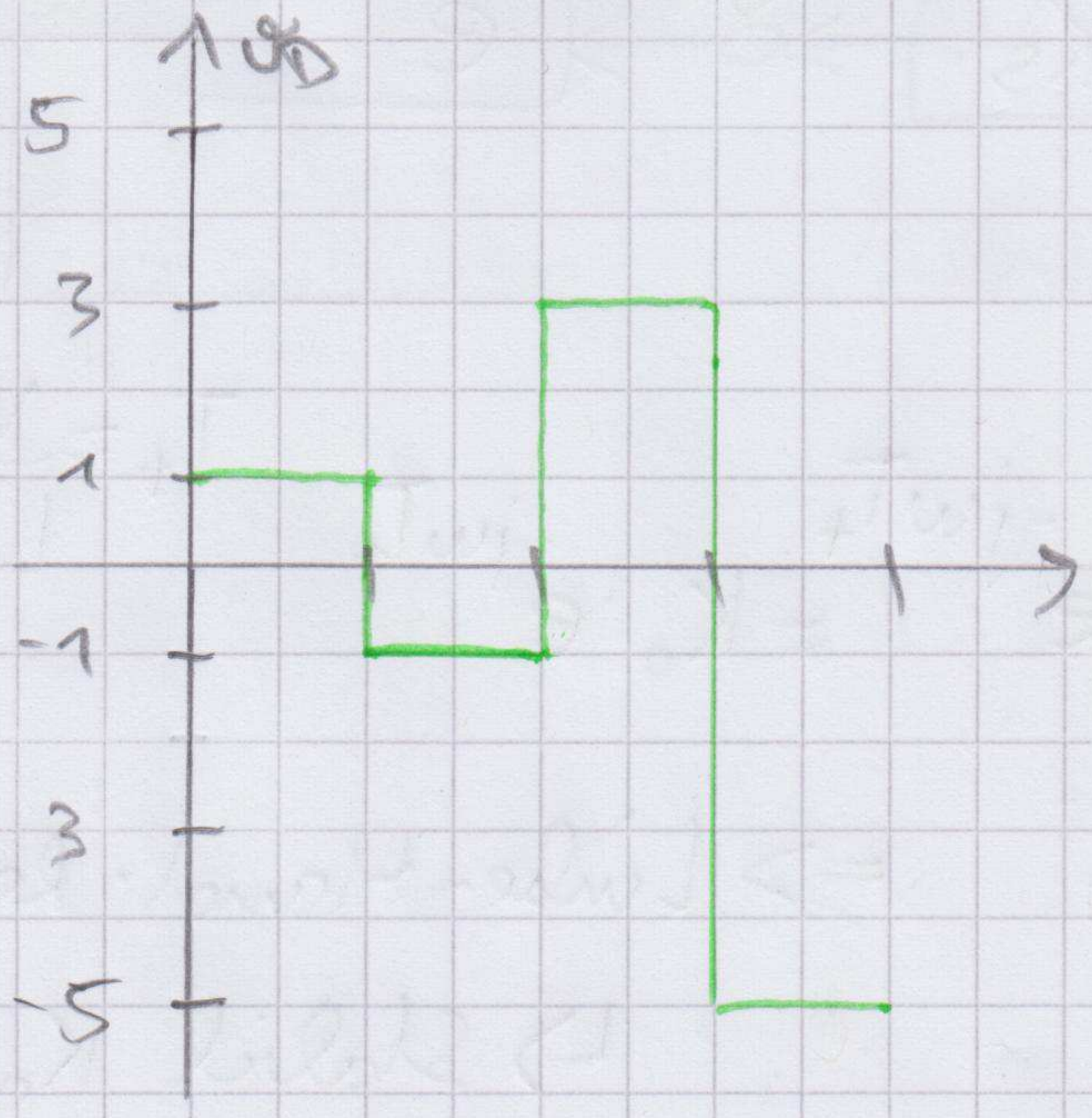
$$F_W = \frac{F_0}{1 + F_0} = \frac{0{,}5 \cdot 1}{1 + 0{,}5 \cdot 1} = \frac{1}{3}$$

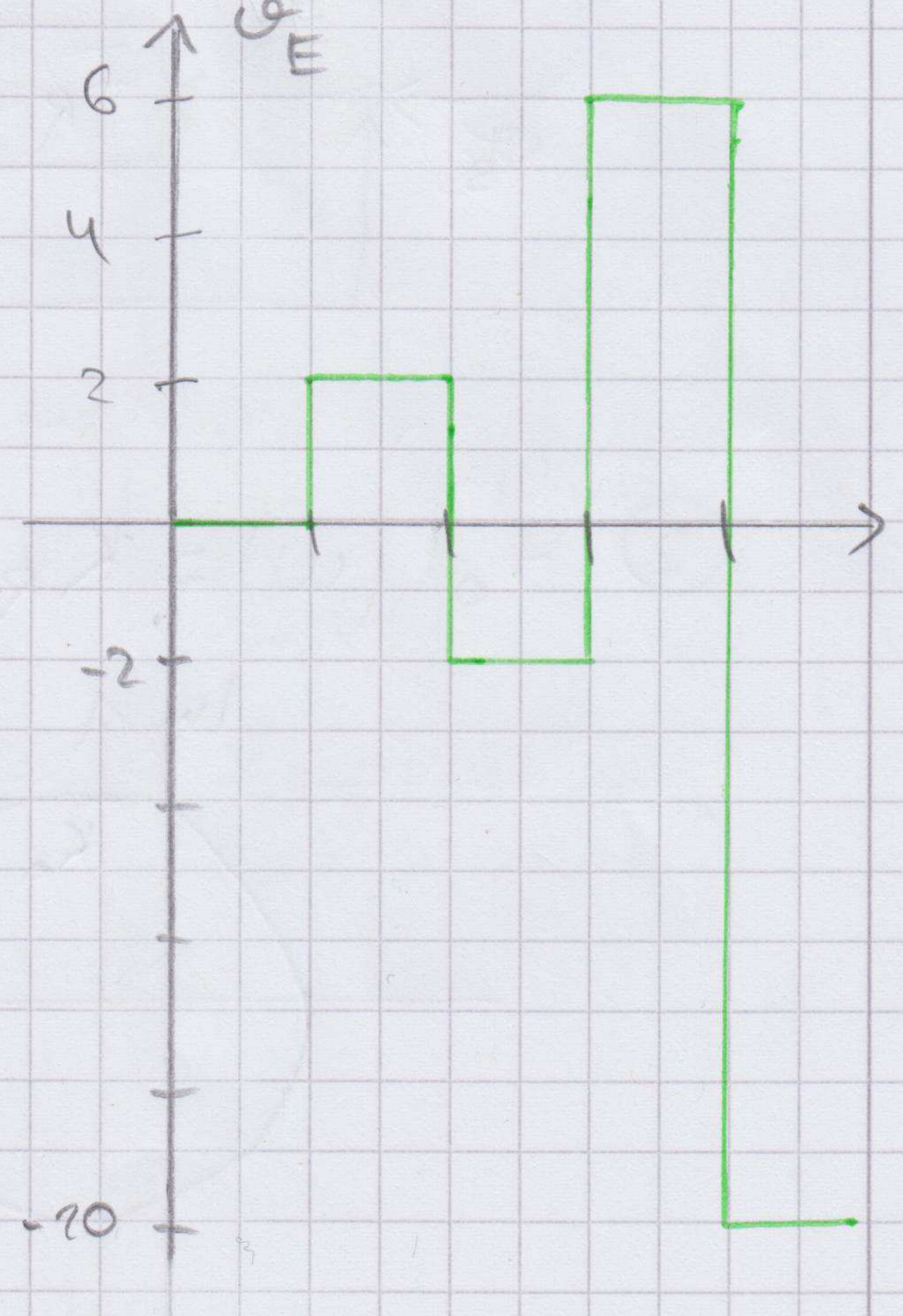
b) $K_0 = K_{0krit} = 1$





c) $K_0 = 2 \cdot K_{0krit}$





(34)

(4)

$$\frac{\vartheta_E(j\omega')}{\vartheta_A(j\omega')} = \frac{e^{-j\omega' T_t}}{1 + K_0 \cdot e^{-j\omega' T_t}} = \frac{e^{-j2\pi\frac{2}{h}\cdot 0{,}5\cdot h}}{1 + 0{,}5\cdot e^{-j2\pi}} = \frac{1}{1{,}5} = \frac{2}{3}$$

$A(\omega') = \frac{2}{3}$; $\varphi(\omega') = -2\pi$