Aufgabe I.2.5

a) $u_{p(t)} = \hat{u} \cdot e^{-at}, \quad a = \frac{2}{t_p}$

allg: $u_{p(t)} = u_p(t - mt_p) = \sum_{m=-\infty}^{\infty} u(t - mt_p)$

$$\underline{c}_\mu = \frac{1}{t_p} \int_0^{t_p} \left(\hat{u} \cdot e^{-at}\right) \cdot e^{-j2\pi\mu f_0 t} \mathrm{dt}$$

$$\underline{c}_\mu = \frac{\hat{u}}{t_p} \int_0^{t_p} e^{-t(a+j2\pi\mu f_0)} \mathrm{dt} \;\rightarrow\; \underline{c}_\mu = \frac{\hat{u}}{t_p} \left[\frac{-1}{(a + j2\pi\mu f_0)} \cdot e^{-(a+j2\pi\mu f_0)t} \right]_0^{t_p}$$

$$\underline{c}_\mu = \frac{-\hat{u}}{at_p + j2\pi\mu} \left[e^{-(a+j2\pi\mu f_0)t_p} - e^0 \right]$$

$$\underline{c}_\mu = \frac{-\hat{u}}{2 + j2\pi\mu} \left[e^{-(2+j2\pi\mu)} - 1 \right] \;,\; e^{-(2+j2\pi\mu)} \;\rightarrow\; e^{-2} \cdot \underbrace{e^{j2\pi\mu}}_{=1}$$

$$\underline{c}_\mu = \frac{\hat{u}}{2 + j2\pi\mu} \left(1 - e^{-2}\right) \;\rightarrow\; \underline{c}_0 = \frac{\hat{u}}{2} \left(1 - e^{-2}\right)$$

b) $\underline{c}_1 = \frac{\hat{u}}{2 + j2\pi} \left(1 - e^{-2}\right) \;\rightarrow\;$ komplex konjugierte $\;\rightarrow\; \frac{2 - j2\pi}{2 - j2\pi}$

$$= \frac{2\hat{u}\left(1 - e^{-2}\right) - j2\pi\hat{u}\left(1 - e^{-2}\right)}{4 + 4\pi^2} \approx (0,03977 - j0,125)\hat{u}$$

$$B_1 = 2|\underline{c}_1| = 2\hat{u}\sqrt{0,03977^2 + 0,125^2} = 0,26\hat{u}$$

$$\Phi_1 = \arg\{\underline{c}_1\} = \arctan\left(\frac{Im\{\underline{c}_1\}}{Re\{\underline{c}_1\}}\right)$$

$$\varphi_1 = \Phi_1 + \frac{\pi}{2} = \arctan\left(\frac{-0,125}{0,03977}\right) + \frac{\pi}{2} \approx 0,308 \;\rightarrow\; \varphi_1 = 17,6^\circ$$

