Aufgabe I.2.7

a) Ungerade ($a_\mu = 0$) , Halbwellensymmetrie ($a_{2k} = 0, b_{2k} = 0 \;\rightarrow$ Nur ungerade Koeffizienten)

b) $$u_{eff}^2 = \frac{1}{t_p}\int_{t_p} u_p^2(t)\mathrm{dt} \;=\; \frac{4}{t_p}\int_0^{\frac{t_p}{4}} u_p^2(t)\mathrm{dt} \;=\; \frac{4}{t_p}\int_0^{\frac{t_p}{4}} \left(\frac{4\hat{u}}{t_p}t\right)^2\mathrm{dt} \;=\; \frac{64\hat{u}^2}{t_p^3}\left[\frac{1}{3}t^3\right]_0^{\frac{t_p}{4}} \;=\; \frac{64\hat{u}^2}{t_p^3}\frac{1}{3}\frac{t_p^3}{64} = \frac{\hat{u}^2}{3}$$

$$\hat{u}_{eff} = \frac{\hat{u}}{\sqrt{3}} \qquad P = \frac{u_{eff}^2}{R} = \frac{\hat{u}^2}{3\Omega}$$

c) $b_\mu \sim \frac{1}{\mu^2}$ Funktion enthält keine Sprünge, aber Knicke

d) $b_{2k} = 0, a_\mu = 0$

$$b_\mu = \frac{8}{t_p}\int_0^{\frac{t_p}{4}} u_p(t)\sin(2\pi\mu f_0 t)\mathrm{dt} \;=\; \frac{8}{t_p}\int_0^{\frac{t_p}{4}} \frac{4\hat{u}}{t_p}t\sin(2\pi\mu f_0 t)\mathrm{dt} \qquad \left(\int x\sin(ax) = \frac{1}{a^2}\sin(ax) - \frac{1}{a}x\cos(ax)\right)$$

$$= \frac{32\hat{u}}{t_p^2}\left[\frac{1}{(2\pi\mu f_0)^2}\cdot\sin(2\pi\mu f_0 t) - \frac{1}{2\pi\mu f_0}t\cos(2\pi\mu f_0 t)\right]_0^{\frac{t_p}{4}}$$

$$= \frac{32\hat{u}}{t_p^2}\left[\frac{1}{4\pi^2\mu^2 f_0^2}\cdot\sin\left(\mu\frac{\pi}{2}\right) - \frac{1}{2\pi\mu f_0}\frac{t_p}{4}\cdot\underbrace{\cos\left(\mu\frac{\pi}{2}\right)}_{0, wenn\;\mu=2k+1}\right]$$

$$= \frac{8\hat{u}}{\mu^2\pi^2}\sin\left(\mu\frac{\pi}{2}\right) \qquad \text{für } \mu = 2k+1$$

$$b_1 = \frac{8\hat{u}}{\pi^2} = B_1 \qquad B_\mu = |b_\mu|$$

e) $$k = \sqrt{\frac{u_{eff}^2 - \overline{u}^2 - u_{1,eff}^2}{u_{eff}^2 - \overline{u}^2}} \qquad \text{mit} \quad u_{eff} = \frac{\hat{u}}{\sqrt{3}}, \qquad \overline{u} = 0 = u_0, \qquad u_{1,eff} = \frac{B_1}{\sqrt{2}} = \frac{8\hat{u}}{\sqrt{2}\pi^2}$$

$$= \sqrt{\frac{\frac{\hat{u}}{3} - \frac{64\hat{u}^2}{2\pi^4}}{\frac{\hat{u}^2}{3}}} = \sqrt{1 - \frac{96}{\pi^4}}$$

$$k_3 = \sqrt{\frac{u_{3,eff}^2}{u_{eff}^2 - \overline{u}^2}} \qquad \rightarrow \quad B_3 = |b_3| = \left|\frac{8\hat{u}}{3^2\pi^2}\sin\left(\frac{3}{2}\pi\right)\right| = \frac{8\hat{u}}{9\pi^2}$$

$$u_{3,eff} = \frac{B_3}{\sqrt{2}} = \frac{8\hat{u}}{9\sqrt{2}\pi^2}$$

$$k_3 = \sqrt{\frac{\left(\frac{8\hat{u}}{9\sqrt{2}\pi^2}\right)^2}{\frac{\hat{u}^2}{3}}} = 0,11$$

