

Aufgabe I.2.8

$$\text{a) } A_u = \frac{U_2}{U_1} = \frac{\frac{1}{j2\pi fC}}{R + \frac{1}{j2\pi fC}} = \frac{1}{1 + j2\pi fRC} = \frac{1}{1 + j2\pi f\tau_1} \quad (\text{wobei } \tau_1 = RC = 0,33\text{ms})$$

$$|A_u(f)| = \sqrt{\frac{1}{1 + 4\pi^2 f^2 \tau_1^2}} \quad \varphi_u(f) = \arctan(2\pi f\tau_1)$$

$$u_e(t) = \underbrace{3V}_{B_0} + \underbrace{4V}_{B_1} \sin(2\pi f_0 t) + \underbrace{5V}_{B_3} \sin(2\pi 3f_0 t)$$

$$u_a = u_e A_0(t)$$

$$\begin{aligned} u_a(t) &= B_0 |A_u(0)| + B_1 |A_u(f_0)| \sin(2\pi f_0 t + \varphi_u(f_0)) + B_3 |A_u(3f_0)| \sin(2\pi 3f_0 t + \varphi_u(3f_0)) \\ &= 3V \cdot 1 + 4V \cdot 0,69 \sin(2\pi f_0 t - 46^\circ) + 5V \cdot 0,31 \sin(2\pi 3f_0 t - 72,2^\circ) \\ &= 3V + 2,77V \sin(2\pi f_0 t - 46^\circ) + 1,53V \sin(2\pi 3f_0 t - 72,2^\circ) \end{aligned}$$

$$\text{b) } U_{a,0eff} = B_{a,0} \quad U_{a,1eff} = \frac{B_{a,1}}{\sqrt{2}} \quad U_{a,3eff} = \frac{B_{a,3}}{\sqrt{2}}$$

$$U_a = \sqrt{B_{a,0}^2 + \frac{B_{a,1}^2}{2} + \frac{B_{a,3}^2}{2}} = \sqrt{3V^2 + \frac{2,77V^2}{2} + \frac{1,53V^2}{2}} = 3,74V$$

$$\text{c) Allgemein in diesem Fall: } I = \frac{U_e}{R + \frac{1}{j2\pi fC}}$$

$$i_1 = \frac{U_e(f_0)}{R + \frac{1}{j2\pi f_0 C}} = \frac{4V}{100\Omega + \frac{1}{j2\pi 500\text{Hz} \cdot 3,3\mu\text{F}}} = 28,8\text{mA} \cdot e^{j43,97^\circ}$$

$$i_3 = \frac{U_e(3f_0)}{R + \frac{1}{j2\pi 3f_0 C}} = \frac{5V}{100\Omega + \frac{1}{j2\pi 3 \cdot 500\text{Hz} \cdot 3,3\mu\text{F}}} = 47,6\text{mA} \cdot e^{j17,82^\circ}$$

$$i_{1,eff} = \frac{|i_1|}{\sqrt{2}} = 20,4\text{mA} \quad i_{3,eff} = \frac{|i_3|}{\sqrt{2}} = 33,7\text{mA} \quad i_{eff} = \sqrt{i_{1,eff}^2 + i_{3,eff}^2}$$

$$P = i_{eff}^2 \cdot R = (i_{1,eff}^2 + i_{3,eff}^2) \cdot R = 0,1527\text{W}$$