Aufgabe I.3.1

a) $U(f) = \int\limits_{-\infty}^{\infty} u(t) e^{-j2\pi ft} \mathrm{dt}$

$$u(t) = \begin{cases} \frac{U_0}{T}(t+T), & -T \leqslant t \leqslant 0 \\ -\frac{U_0}{T}(t-T), & 0 \leqslant t \leqslant T \\ 0, & sonst \end{cases}$$

$$U(f) = \int\limits_{-T}^{0} \frac{U_0}{T}(t+T)e^{-j2\pi ft}\mathrm{dt} + \int\limits_{0}^{T} -\frac{U_0}{T}(t-T)e^{-j2\pi ft}\mathrm{dt} \quad \text{(auch mit Symmetrieeigenschaften lösbar)}$$

$$= \frac{U_0}{T}\left(\int\limits_{-T}^{0} \left(te^{-j2\pi ft} + Te^{-j2\pi ft}\right)\mathrm{dt} - \int\limits_{0}^{T} \left(te^{-j2\pi tf} + Te^{-j2\pi ft}\right)\mathrm{dt} \right)$$

Additionstheoreme: $\int e^{ax}\mathrm{dx} = \frac{1}{a}e^{ax} \qquad \int xe^{ax}\mathrm{dx} = (ax-1)\frac{1}{a^2}e^{ax}$

$$= \frac{U_0}{T}\left(\left[(-j2\pi ft - 1)\frac{1}{(j2\pi f)^2}e^{-j2\pi ft} - \frac{T}{j2\pi f}e^{-j2\pi ft}\right]_{-T}^{0} - \left[(-j2\pi ft - 1)\frac{1}{(2j\pi f)^2}e^{-j2\pi ft} + \frac{T}{j2\pi f}e^{-j2\pi ft}\right]_{0}^{T} \right)$$

$$= \frac{U_0}{T}\left(\left[-\frac{1}{(j2\pi f)^2} - \frac{T}{j2\pi f} - (j2\pi fT - 1)\frac{1}{(j2\pi f)^2}e^{j2\pi fT} + \frac{T}{j2\pi f}e^{j2\pi fT}\right] - \left[(-j2\pi fT - 1)\frac{1}{(j2\pi f)^2}e^{-j2\pi fT} + \frac{T}{j2\pi f}e^{-j2\pi fT} + \frac{1}{(j2\pi f)^2} - \frac{T}{j2\pi f}\right] \right)$$

$$= \frac{U_0}{T}\left[-\frac{2}{(j2\pi f)^2} - \frac{j2\pi fT}{(j2\pi f)^2}e^{j2\pi fT} + \frac{j2\pi fT}{(j2\pi f)^2}e^{-j2\pi fT} + \frac{1}{(j2\pi f)^2}e^{j2\pi fT} + \frac{1}{(j2\pi f)^2}e^{-j2\pi fT} + \frac{T}{j2\pi f}e^{j2\pi fT} - \frac{T}{j2\pi f}e^{-j2\pi fT} \right]$$

$$= \frac{U_0}{T}\left[-\frac{2}{(j2\pi f)^2} + \frac{1}{(j2\pi f)^2}e^{j2\pi fT} + \frac{1}{(j2\pi f)^2}e^{-j2\pi fT} \right]$$

$$= \frac{U_0}{T}\frac{1}{4\pi^2 f^2}\left[2 - \left(e^{j2\pi fT} + e^{-j2\pi fT}\right) \right]$$

Additionstheorem: $\cos(x) = \frac{e^{jx} + e^{-jx}}{2}$

$$U(f) = \frac{U_0}{T} \frac{1}{4\pi^2 f^2} \bigg(2 - 2\cos(2\pi f T)\bigg)$$

Additionstheorem: $\sin(\alpha)^2 = \frac{1}{2} - \frac{1}{2}\cos(2\alpha)$

$$= \frac{U_0}{T} \frac{1}{\pi^2 f^2} \sin(\pi f T)^2 \qquad \bigg| \frac{T}{T}$$

$$= U_0 \cdot T \cdot \frac{\sin(\pi f T)^2}{\pi^2 f^2 T^2} = U_0 \cdot T \cdot \mathrm{si}(\pi f T)^2$$

