18.4.12

Übungsblatt 4

1) ①

$$F_w(s) = \frac{K_p \cdot \frac{3}{1+2s}}{1 + K_p \frac{3}{1+2s}} = \frac{3K_p}{2s+1+3K_p} = \frac{\frac{3K_p}{1+3K_p}}{\frac{2}{1+3K_p} \cdot s + 1}$$

$$= \frac{K_{RK}}{T_{RK} \cdot s + 1}$$

$$F_z(s) = \frac{-\frac{3}{1+2s}}{1 + K_p \cdot \frac{3}{1+2s}} = \frac{-3}{2s+1+3K_p} = \frac{-\frac{3}{1+3K_p}}{\frac{2}{1+3K_p} \cdot s + 1}$$

② $T_{ein} = 3 \cdot T \Rightarrow 3 \cdot T_{RK} = 3 \Rightarrow T_{RK} = 1$

$$\Rightarrow \frac{2}{1+3K_p} = 1 \Rightarrow 1 + 3K_p = 2 \Rightarrow K_p = \frac{1}{3}$$

③ $x(t \to \infty) = \lim_{s \to 0} (s \cdot F_w(s) \cdot w(s))$

$$= \lim_{s \to 0} \left(s \cdot F_w(s) \cdot \frac{1}{s}\right) = K_{RK} = 0{,}9$$

$$\Rightarrow \frac{3 \cdot K_p}{1 + 3 \cdot K_p} = 0{,}9 \Rightarrow K_p = 3$$

$$\lim_{s \to 0} F_{xd}(s) = \lim_{s \to 0} \frac{xd(s)}{w(s)} = 0{,}1$$

④ $\frac{xd(s)}{w(s)} = \frac{1}{1 + F_o(s)} = 1 - F_w(s) = 1 - \frac{F_o(s)}{1+F_o(s)}$

$$= \frac{1+F_o(s)}{1+F_o(s)} - \frac{F_o(s)}{1+F_o(s)}$$

$$xd(t \to \infty) = \lim_{s \to 0} s \cdot (1 - F_w(s)) \cdot \frac{2}{s}$$

$$= \ldots$$

$$= \lim_{s \to 0} 2\left(1 - \frac{6/7}{\frac{2}{7} \cdot s + 1}\right) = \frac{2}{7}$$

$$y(\infty) = \lim_{s \to 0} \left(s \cdot F_y \cdot \frac{2}{s}\right) = \lim_{s \to 0} \left(2 \cdot \frac{F_R}{1 + F_0(s)}\right) = K \cdot x_d(\infty)$$

$$= \frac{4}{7}$$

(5) $z = \sigma(t)$, $x_d(\infty) = ?$, $y(\infty) = ?$

$x_d(\infty) = -x(\infty)$

$$x_d(\infty) = \lim_{s \to 0} \left(-F_z(s)\right) = \lim_{s \to 0} -\left(\frac{-\frac{3}{7}}{\frac{2}{7} \cdot s + 1}\right) = \frac{3}{7}$$

$$y(\infty) = K_p \cdot x_d(\infty) = \frac{6}{7}$$

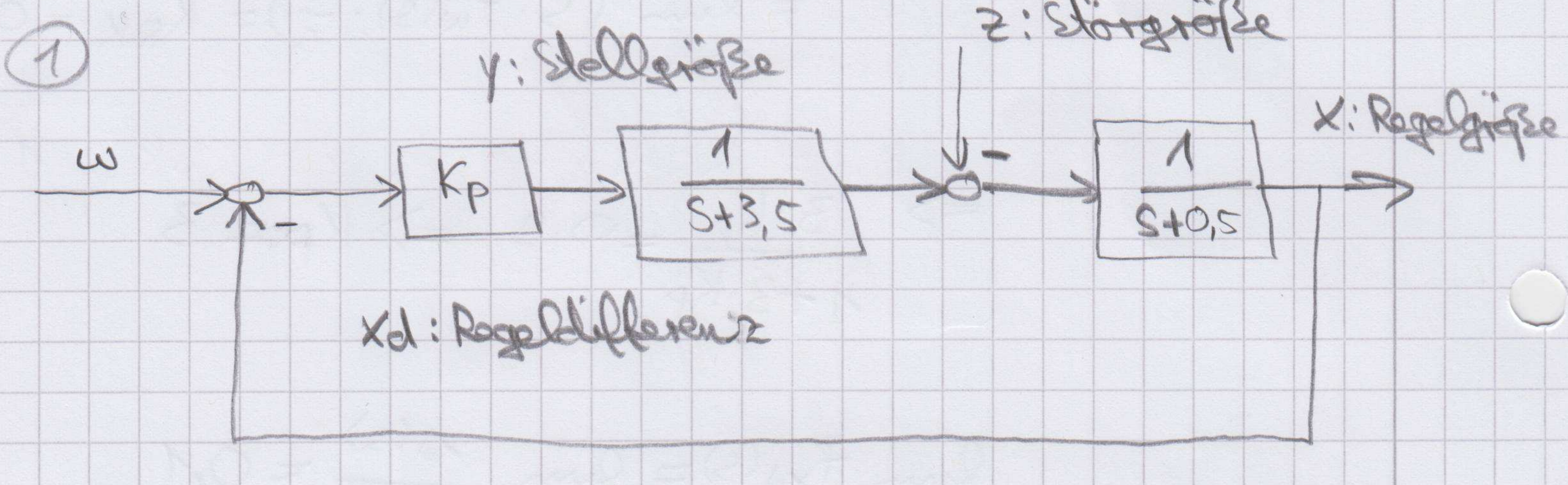
(6) Superposition:

$$x_d(\infty) = \frac{2}{7} + \frac{3}{7} = \frac{5}{7}$$

$$y(\infty) = \frac{4}{7} + \frac{6}{7} = \frac{10}{7}$$

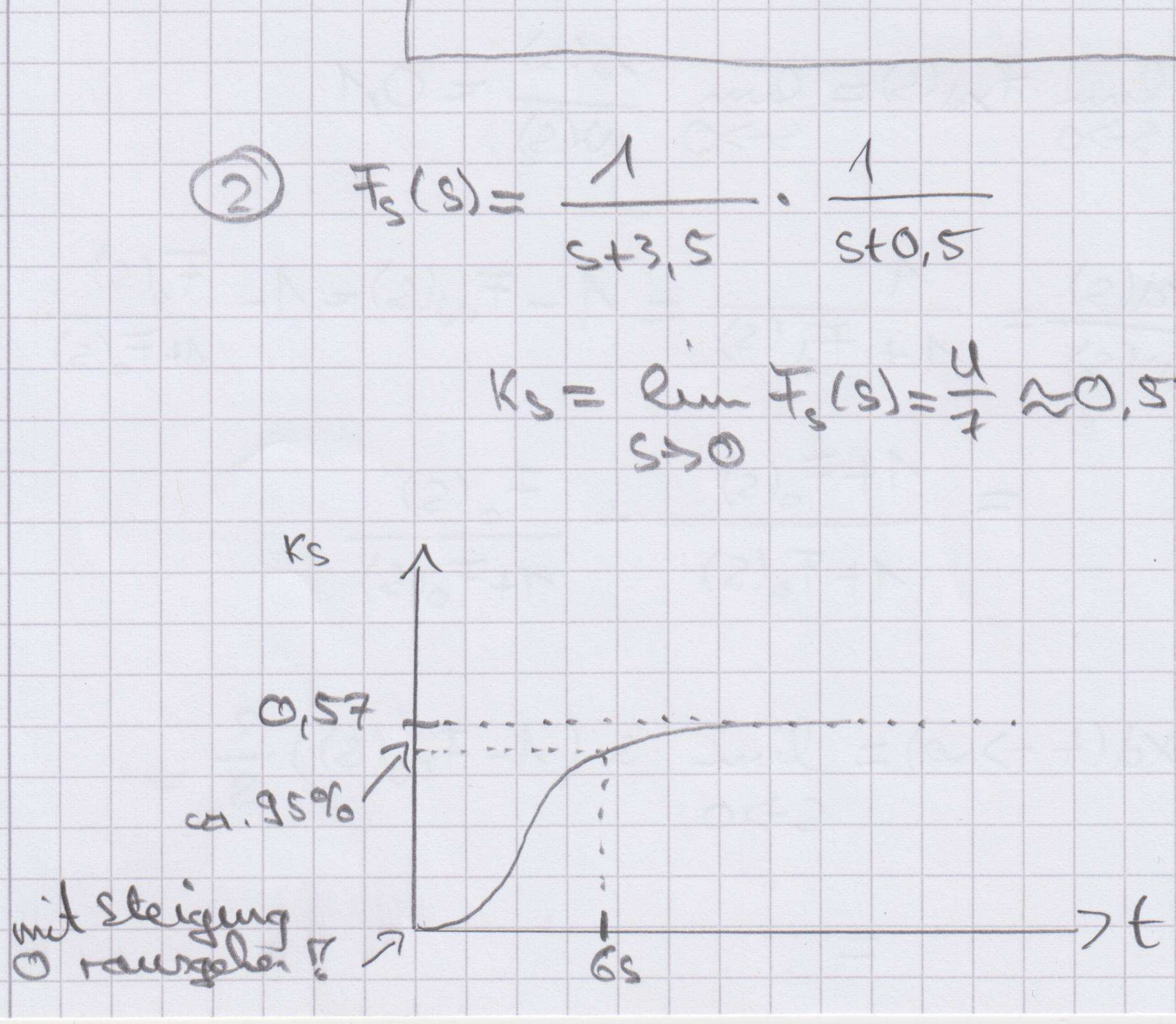
2)

(1)



(2) $$F_s(s) = \frac{1}{s+3,5} \cdot \frac{1}{s+0,5}$$

$$K_s = \lim_{s \to 0} F_s(s) = \frac{4}{7} \approx 0,57$$



③ $$F_w(s) = \frac{K_p \cdot \frac{1}{s+3,5} \cdot \frac{1}{s+0,5}}{1 + K_p \cdot \frac{1}{s+3,5} \cdot \frac{1}{s+0,5}} = \frac{K_p}{s^2+4s+1,75+K_p}$$

$$F_z(s) = \frac{-\frac{1}{s+0,5}}{1 + K_p \cdot \frac{1}{s+3,5} \cdot \frac{1}{s+0,5}} = \frac{-(s+3,5)}{s^2+4s+1,75+K_p}$$

④ $\underline{w = \sigma(t)}$: $$x_d(\infty) = \lim_{s \to 0} (1 - F_w(s)) = 1 - \frac{K_p}{1,75+K_p} =$$

$$= 1 - \frac{1,25}{3} = \frac{7}{12}$$

$$y(\infty) = K_p \cdot x_d(\infty) = \frac{35}{48}$$

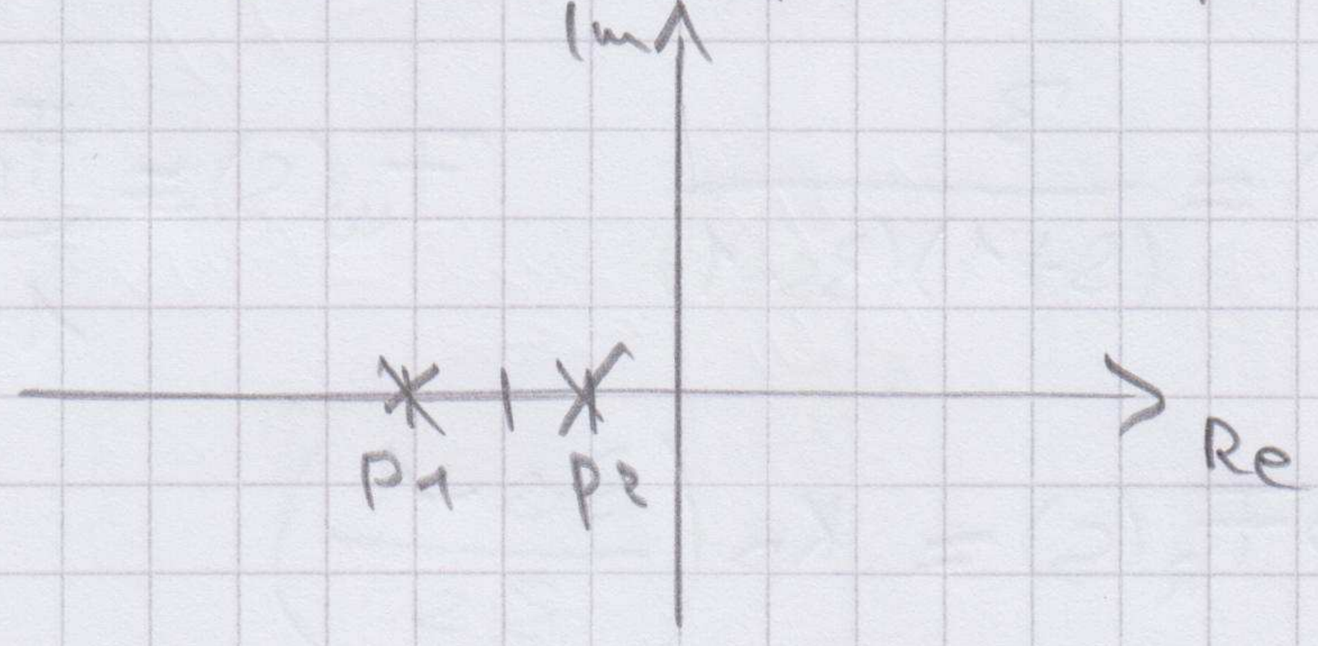
$\underline{z = \sigma(t)}$: $$x_d(\infty) = \lim_{s \to 0} (-F_z(s)) = \frac{3,5}{3} = \frac{7}{6}$$

$$y(\infty) = K_p \cdot x_d(\infty) = \frac{5}{4} \cdot \frac{7}{6} = \frac{35}{24}$$

23.4.2012

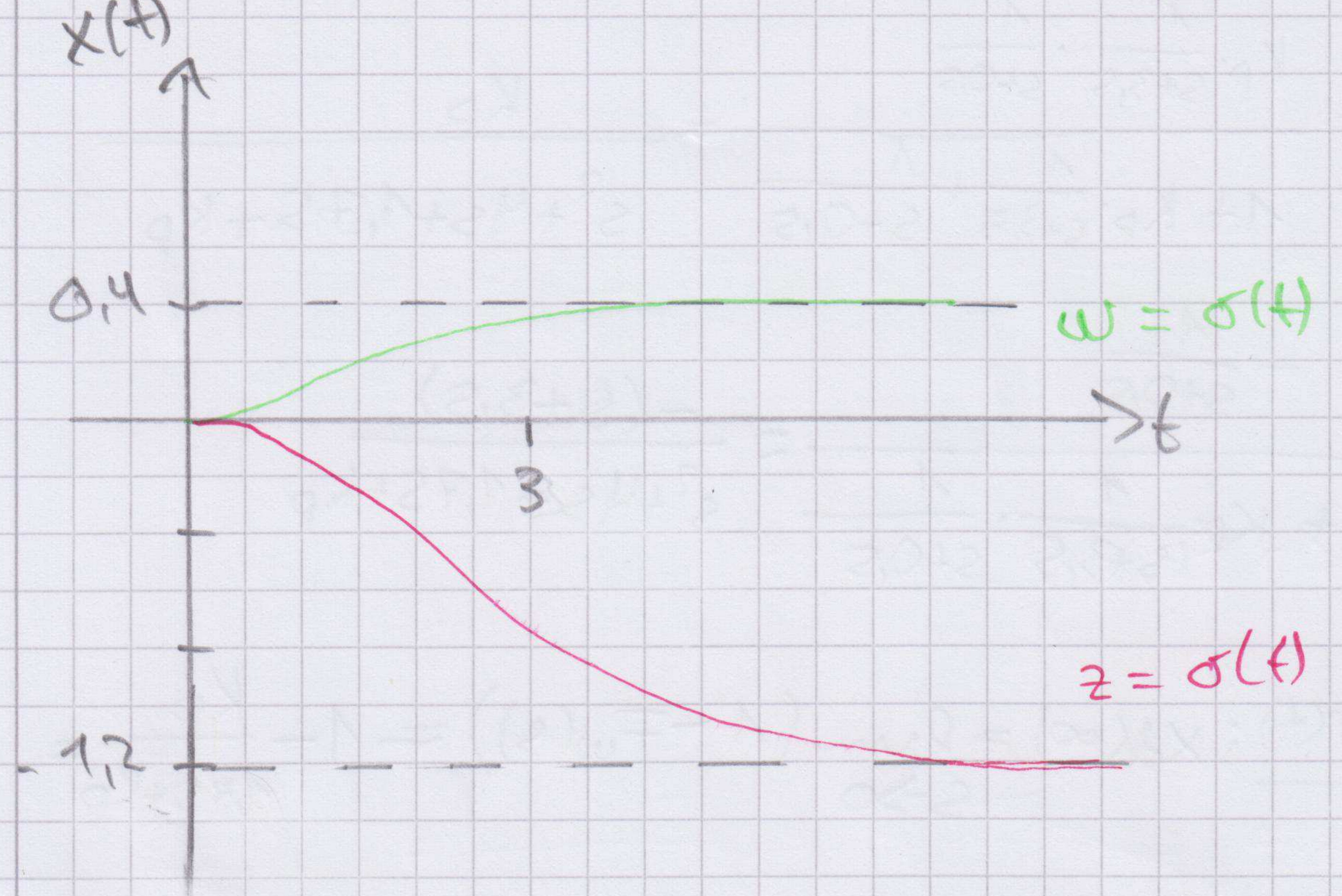
⑤ $N(s) = s^2 + 4s + 3$

$$p_{1/2} = -2 \pm \sqrt{\frac{16}{4} - 3} \Rightarrow p_1 = -3,\ p_2 = -1$$



⑥ $$w = \sigma(t) \Rightarrow x(\infty) = \lim_{s \to 0} F_s(s) = \frac{5}{12} = 0,42$$

$$z = \sigma(t) \Rightarrow x(\infty) = \lim_{s \to 0} F_z(s) = -\frac{7}{6} = -1,16$$



⑦ $N(s) = s^2 + 4s + 1{,}75 + K_p$

$= s^2 + 2 \cdot D \cdot \omega_0 \cdot s + \omega_0^2$

$\Rightarrow 2 \cdot D \cdot \omega_0 = 4 \Rightarrow \omega_0 = \frac{4}{\sqrt{2}}$

$\Rightarrow 1{,}75 + K_p = \omega_0^2 = 8$

$\Rightarrow K_p = 6{,}25$

3)

$F_R(s) = K_p + \frac{K_I}{s} = K_p\left(1 + \frac{1}{\frac{K_p}{K_I} \cdot s}\right) = K_p\left(1 + \frac{1}{T_n \cdot s}\right)$

$= K_p\left(\frac{T_n \cdot s + 1}{T_n \cdot s}\right)$

① $F_s(s) = \frac{3}{(s+1)(3s+1)}$ $\qquad F_w(s) = \frac{F_R \cdot F_s}{1 + F_R \cdot F_s}$

$\Rightarrow F_R(s) = K_p\left(\frac{3 \cdot s + 1}{3 \cdot s}\right)$

② $F_0(s) = F_R \cdot F_s = K_p\left(\frac{\cancel{3s+1}}{\cancel{3}s}\right) \cdot \left(\frac{\cancel{3}}{(s+1)(\cancel{3s+1})}\right)$

$= K_p \cdot \frac{1}{s(s+1)} = \frac{Z_0}{N_0}$

$N_{RK}(s) = Z_0 + N_0 = s^2 + s + K_p = b_2 \cdot s^2 + b_1 \cdot s + b_0$

stabil für $b_i > 0 \Rightarrow K_p > 0$

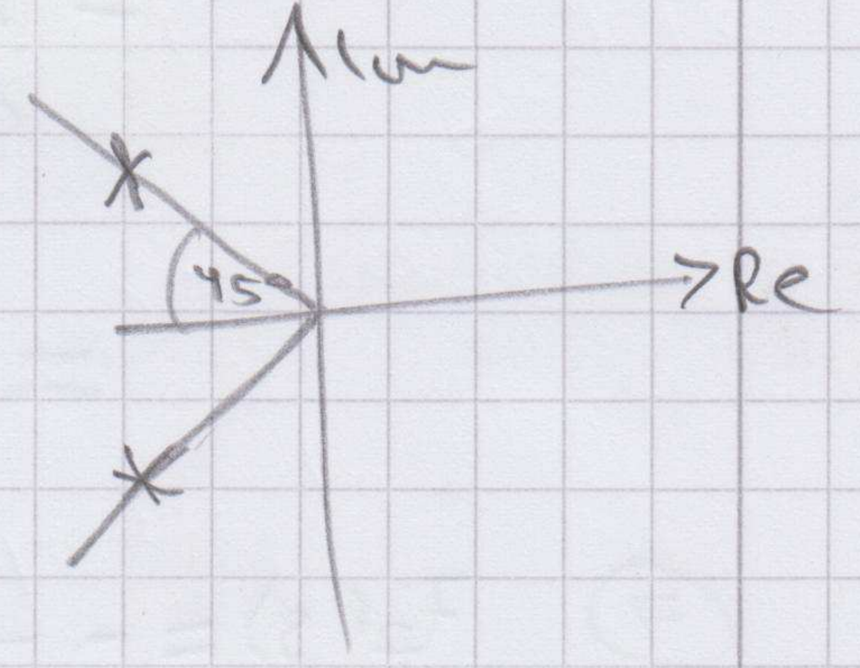
③ $p_{1,2} = -\frac{1}{2} \pm \sqrt{\frac{1}{4} - K_p}$

Doppelpol: $K_{p1} = 0{,}25 \Rightarrow p_{1,2} = -\frac{1}{2}$

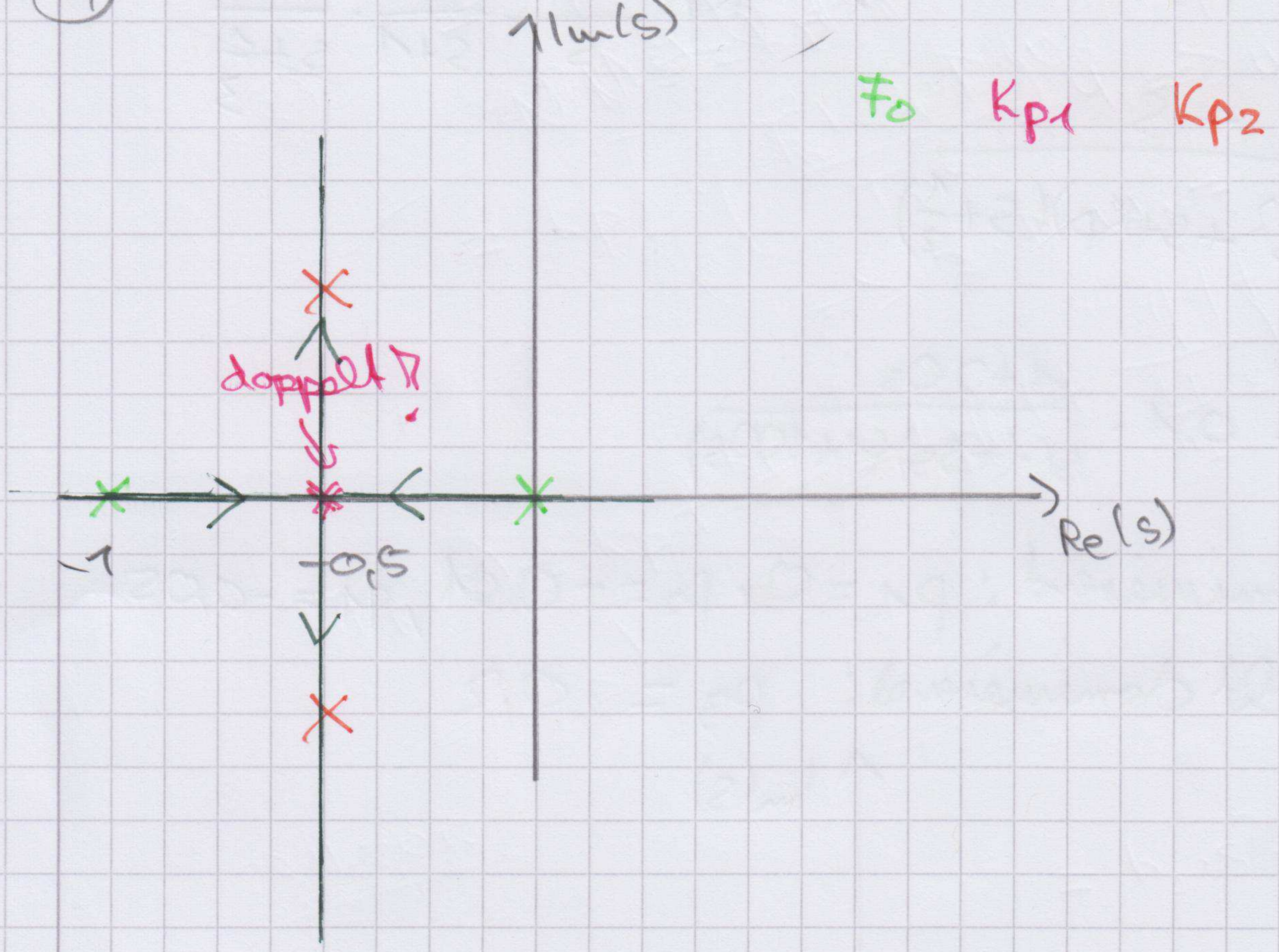
Dämpfung $D = \frac{1}{\sqrt{2}}$: $p_{1,2} = -\omega_0 \cdot D \pm j \cdot \omega_0 \sqrt{1-D^2}$

$= -\omega_0 \left(\frac{1}{\sqrt{2}} \pm j \frac{1}{\sqrt{2}} \right)$

$\Rightarrow K_{p2} = 0{,}5$



④



⑤ $K_{p2} = 0{,}5$

$\lim_{s \to 0} (1 - F_w(s)) \cdot w_0 = 0$

$F_w(s) = \frac{\frac{0{,}5}{s(s+1)}}{1 + \frac{0{,}5}{s(s+1)}} = \frac{0{,}5}{s^2 + s + 0{,}5}$

$y(\infty) = \lim_{s \to 0} \left(\frac{F_R(s)}{1 + F_R(s) \cdot F_S(s)} \right) \cdot 2 = \frac{x(\infty)}{K_S} = \frac{2}{3}$

$x_d(0) = \lim_{s \to \infty} s \cdot (1 - F_w(s)) \frac{2}{s} = \lim_{s \to \infty} 2 \cdot (1 - F_w(s)) = 2$

$y(0^+) = \lim_{s \to \infty} 2 \cdot \left(\frac{F_R(s)}{1 + F_R(s) \cdot F_S(s)} \right) = 1$

⑥ $w(t) = 3 \cdot t \cdot \sigma(t)$

$$x_d(\infty) = \lim_{s\to 0} s \cdot (1 - F_w(s)) \cdot \frac{3}{s^2} = \lim_{s\to 0} \frac{3}{s}(1 - F_w(s))$$

$$= \lim_{s\to 0} \frac{3}{s}\left(\frac{s^2+s+\cancel{0,5-0,5}}{s^2+s+0,5}\right) = \lim_{s\to 0} 3 \cdot \left(\frac{s+1}{s^2+s+0,5}\right)$$

$$= 6$$

⑦ $$F_Z(s) = -\frac{F_S(s)}{1+F_S(s)+F_R(s)} = \frac{\frac{1}{s+1} \cdot \frac{1}{s+\frac{1}{3}}}{1 + K_p \frac{\cancel{s+\frac{1}{3}}}{s} \cdot \frac{1}{s+1} \cdot \frac{1}{\cancel{s+\frac{1}{3}}}}$$

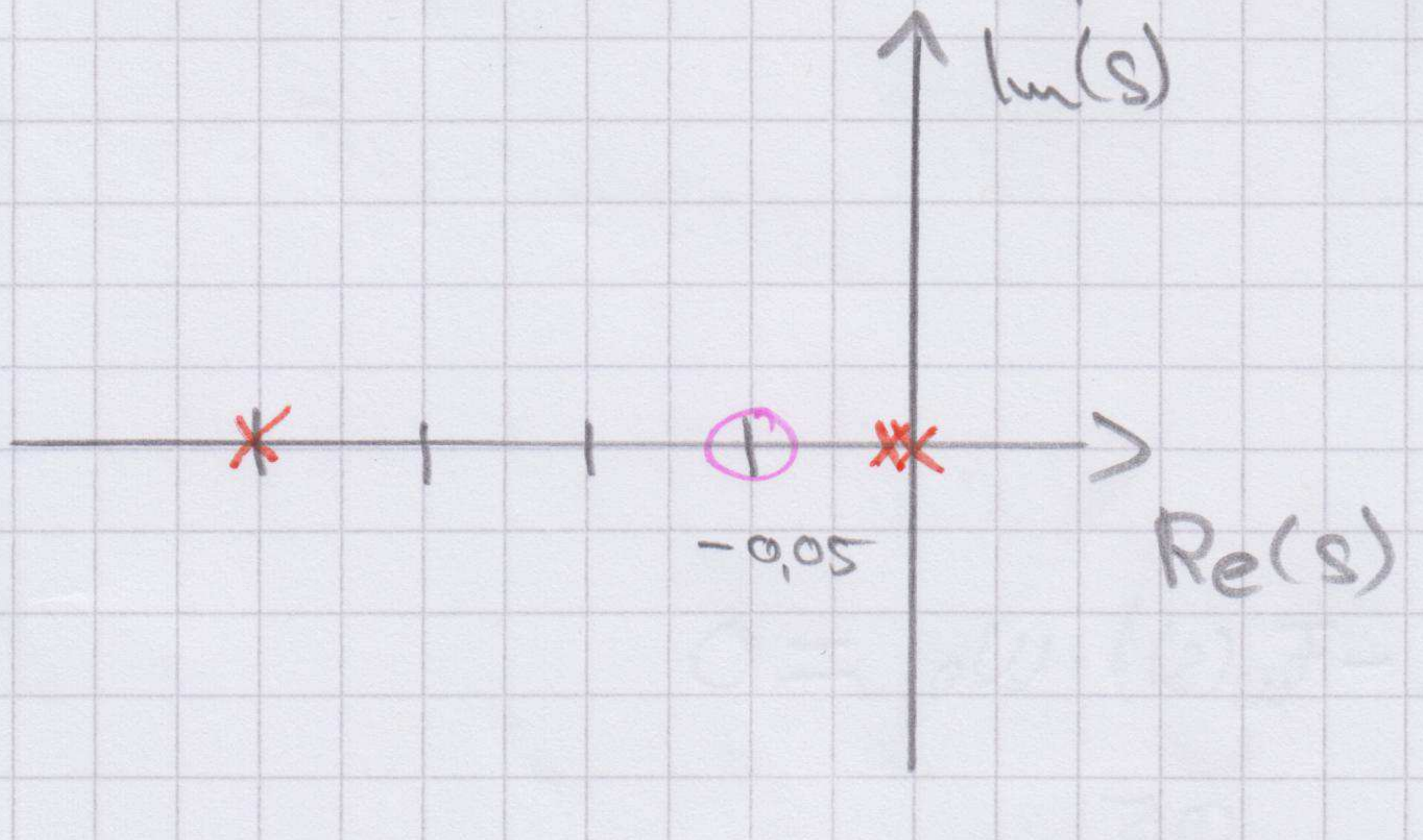
$$= \frac{-s}{(s^2+s+K_p)(s+\frac{1}{3})}$$

25.4.2012

4) ① $F_s(s) = 0,1 \cdot \frac{1+20s}{s(1+5s)(1+100s)}$

=> Dominierend: $p_1 = 0$, $p_2 = -0,01$, $q_1 = -0,05$

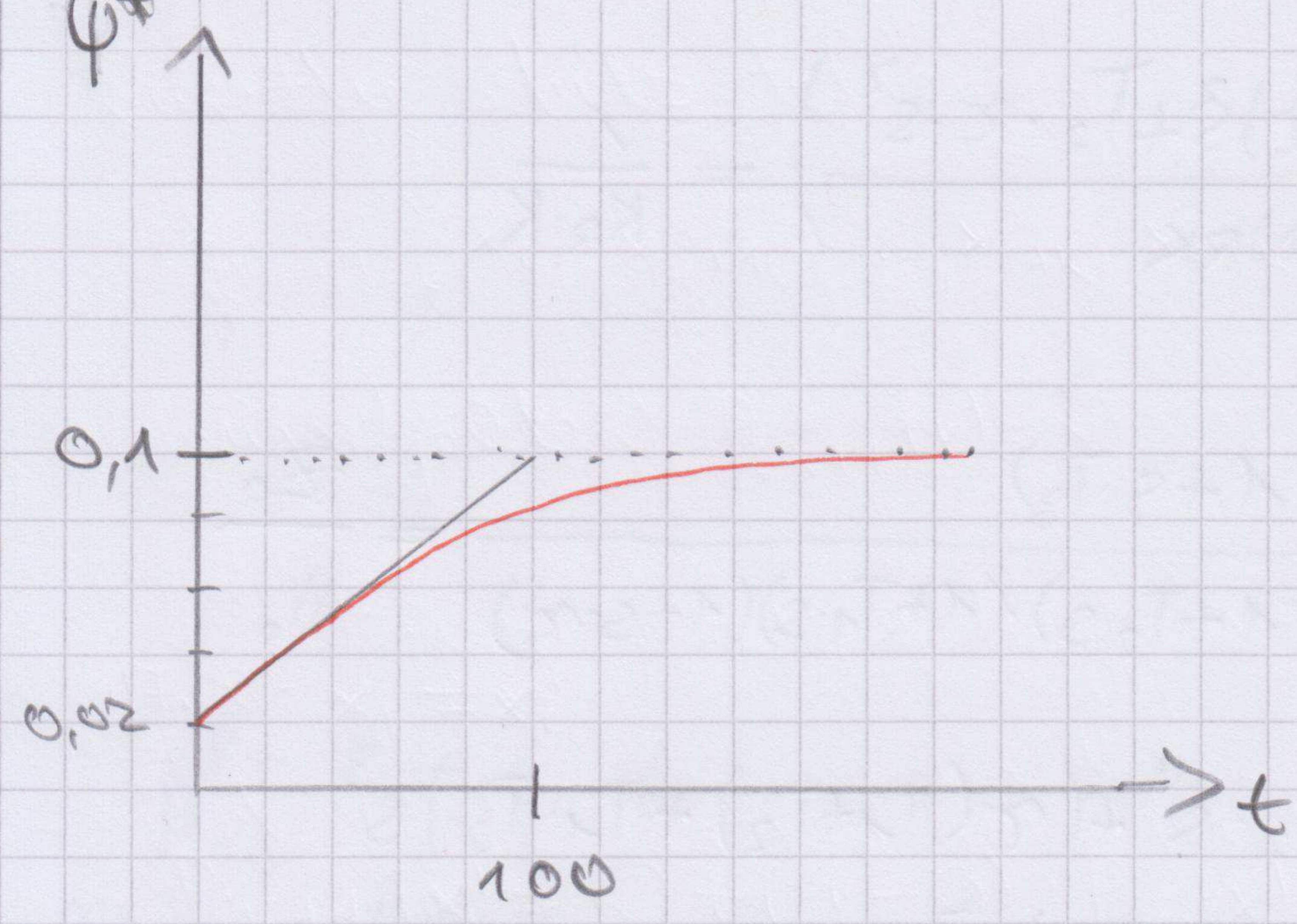
Nicht dominierend: $p_3 = -0,2$



② $F_s^*(s) = 0,1 \cdot \frac{1+20s}{s(1+100s)}$

Anfang: $\lim_{s\to\infty} (s \cdot F_s^*(s) \cdot 1) = 0,2$

Ende: $\lim_{s\to 0} (s \cdot F_s^*(s) \cdot 1) = 0,1$



(3) $F_{RH}(s) = \dfrac{1}{1+\tau\cdot s}$ damit nicht dominierend

$\hookrightarrow \tau < 10$

(4) $F_0(s) = K_K \cdot K \cdot \dfrac{1+s\cdot T_3}{s(1+s\cdot T_2)(1+s\cdot\tau)}$

$\Rightarrow p_1 = 0 \; ; p_2 = -0{,}01 \; ; p_3 = -0{,}1 \; ; q_1 = -0{,}05$

$N_{RK} = Z_0 + N_0 = \tau T_2 \cdot s^3 + (T_2+\tau)s^2 + (1+K_R\cdot K + T_3)\cdot s$
$\quad + K_R \cdot K$
$\quad = (s^3 + 0{,}11 s^2 - 0{,}0028 s + 0{,}00009)\cdot 1000$

Nullstelle weiterhin bei $q_1 = -0{,}05$

$p_{1/2} = -0{,}01 \pm j\,0{,}03 \; ; \; p_3 = -0{,}09$

(5) a) $x_d(\infty) = \lim\limits_{s\to 0} s\cdot\left(1 - F_w(s)\right)\cdot 1 = 0$

$F_w = \dfrac{F_0}{1+F_0} = \dfrac{Z_0}{Z_0+N_0} = \dfrac{1{,}8s+0{,}09}{N_{RK}}$

b) $x_d(\infty) = \lim\limits_{s\to 0} s\cdot(1-F_w(s))\dfrac{1}{s^2}$

$\lim\limits_{s\to 0} \dfrac{1}{s}\left(\dfrac{\cancel{K_R\cdot K} + s(1+\cancel{T_3\cdot K\cdot K_R}) + (T_2+\tau)\cdot s^2}{N_{RK}} + \dfrac{\tau\cdot T_2\cdot s^3 - \cancel{K K_R} - \cancel{K\cdot K_R\cdot T_3\cdot s}}{N_{RK}} \right)$

$$= \lim_{s\to 0} \frac{1}{s}\left(\frac{s+(T_2+\tau)s^2+T_2\cdot\tau\cdot s^3}{N_{RK}}\right) = \frac{1}{K_R\cdot K}$$

6 $$F_0(s) = K_R\cdot K\cdot\frac{(1+s\cdot T_3)}{s(1+T_2\cdot s)\cdot(1+T_1\cdot s)(1+s\cdot\tau)} = \frac{Z_0}{N_0}$$

$$\begin{aligned} N_{RK} &= \tau\cdot T_1\cdot T_2\cdot s^4+(\tau(T_1+T_2)+T_1\cdot T_2)s^3 \\ &\quad +(\tau+T_1+T_2)s^2+(K\cdot K_R\cdot T_3+1)s+K\cdot K_R \\ &= b_4 s^4+b_3 s^3+b_2 s^2+b_1 s+b_0 \end{aligned}$$

Hurwitz Bedingungen:

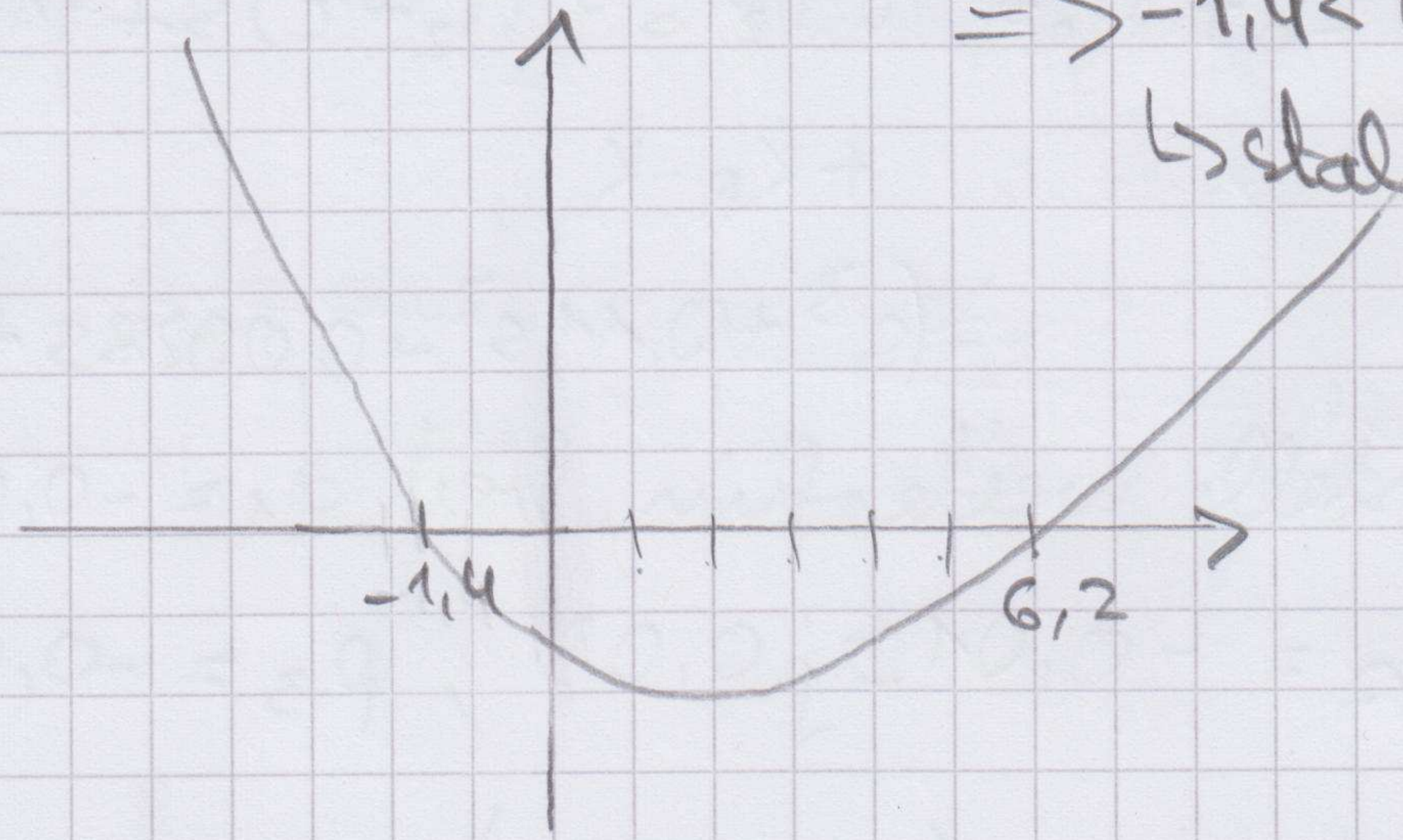
$b_0>0$: $K_R>0$

$b_1>0$: $(2K_R+1)>0 \Rightarrow K_R>-0{,}5$

$b_1 b_2 b_3 - b_0 b_3^2 - b_1^2 b_4 > 0$: $K_R^2-4{,}81K_R-8{,}66<0$

$\Rightarrow -1{,}4<K_R<6{,}2$

↳ stabil für $0<K_R<6{,}4$



5) 1 Ruhelage : $\dddot{x}=\ddot{x}=\dot{x}=0$

$\Rightarrow x_0^3=4\cdot w_0 \Rightarrow w_0=\frac{x_0^3}{4}$

a) Variable Größe : $\dddot{x}, \ddot{x}, \dot{x}, x, w$

b) Linearisierung :

$$64\cdot\underbrace{\frac{\partial\dddot{x}}{\partial\dddot{x}}\Big|_{BP}}_{=1}\Delta\dddot{x}+16(a+1)\cdot x\Big|_{BP}\cdot\Delta\ddot{x}+16(a+1)\dot{x}\Big|_{BP}\cdot\Delta x+12(a-1)\cdot\Delta\dot{x}+3x^2\Big|_{BP}\cdot\Delta x$$

$$64 \cdot \dddot{\Delta x} + 16(a+1)\cdot x_0 \cdot \ddot{\Delta x} + 12(a-1)\cdot \dot{\Delta x} + 3x_0^2 \cdot \Delta x = 4 \cdot \Delta w$$

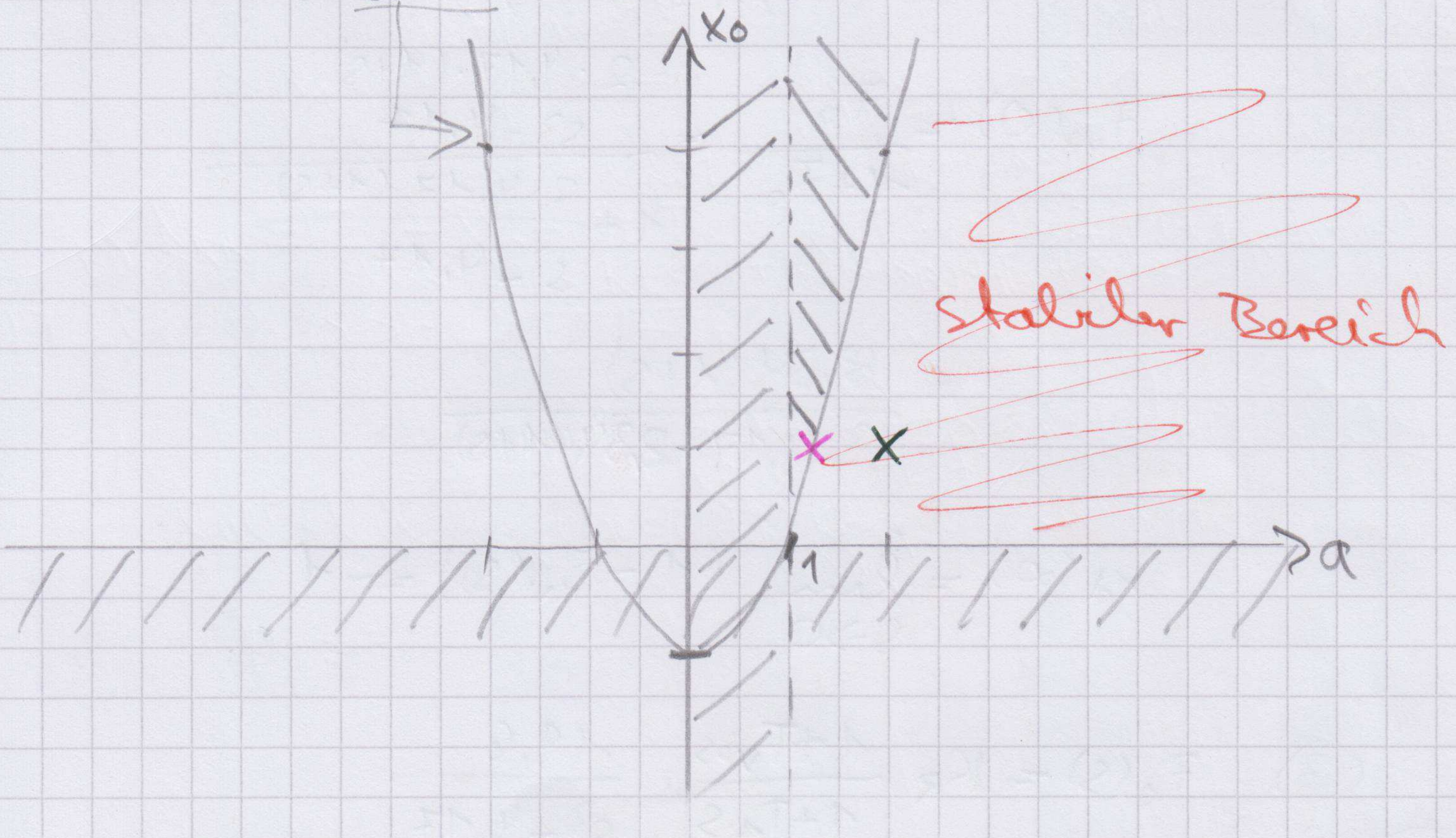
(2) $\Delta x(s) \cdot (64 \cdot s^3 + 16(a+1)\cdot x_0 \cdot s^2 + 12(a-1)\cdot s + 3x_0^2) = 4\Delta w$

$$F(s) = \frac{\Delta x}{\Delta w} = \frac{4}{64s^3 + 16(a+1)x_0 \cdot s^2 + 12(a-1)\cdot s + 3x_0^2}$$

- $b_0 > 0 : x_0 \neq 0$
- $b_1 > 0 : a > 1$
- $b_2 > 0 : 16(a+1)\cdot x_0 > 0 : x_0 > 0$
- $b_1 \cdot b_2 - b_0 \cdot b_3 > 0 : 16(a+1)\cdot x_0 \cdot 12(a-1) - 64 \cdot 3x_0^2 > 0$

$\Rightarrow (a+1)(a-1) - x_0 > 0$

$\Rightarrow a^2 - 1 > x_0$



$\rightarrow$ • stabil für $x_0 = 1, a = 2$

$\rightarrow$ • grenzstabil $x_0 = 1, a = \sqrt{2}$

• $N_{RK} = 64s^3 + 16(\sqrt{2}+1)s^2 + 12(\sqrt{2}-1)s + 3$

$\Rightarrow p_{1,2} = \pm j\omega$

$p_3 = ?$

Koeffizientenvergleich

$\hookrightarrow (s^2 + \omega^2)(s - p_3) \Rightarrow p_{1,2} = \pm j\, 0{,}28$

$p_3 = -0{,}604$

6) 1 a) $F_S(s) = K_R \cdot 19{,}6 \cdot \dfrac{\frac{1}{s^2}}{1 - 4{,}17\frac{1}{s^2}} = K_R \cdot 19{,}6 \cdot \dfrac{1}{s^2 - 4{,}17}$

$N_{RK} = s^2 - 4{,}17 + 19{,}6 \cdot K_R$

$\Rightarrow b_1 = 0 \Rightarrow$ System instabil!

b) $F_o(s) = K_R(1+s) \cdot 19{,}6 \cdot \dfrac{1}{s^2 - 4{,}17}$

$N_{RK} = s^2 + 19{,}6 \cdot K_R \cdot s + 19{,}6 K_R - 4{,}17$

$\Rightarrow b_0 > 0: 19{,}6 K_R - 4{,}17 > 0 \Rightarrow K_R > \dfrac{4{,}17}{19{,}6} = K_{R\,krit}$

$b_1 > 0: K_R > 0$

2 $N_{RK}\left(K_R = K_{R\,krit} = \dfrac{4{,}17}{19{,}6}\right) = s^2 + 4{,}17 s$

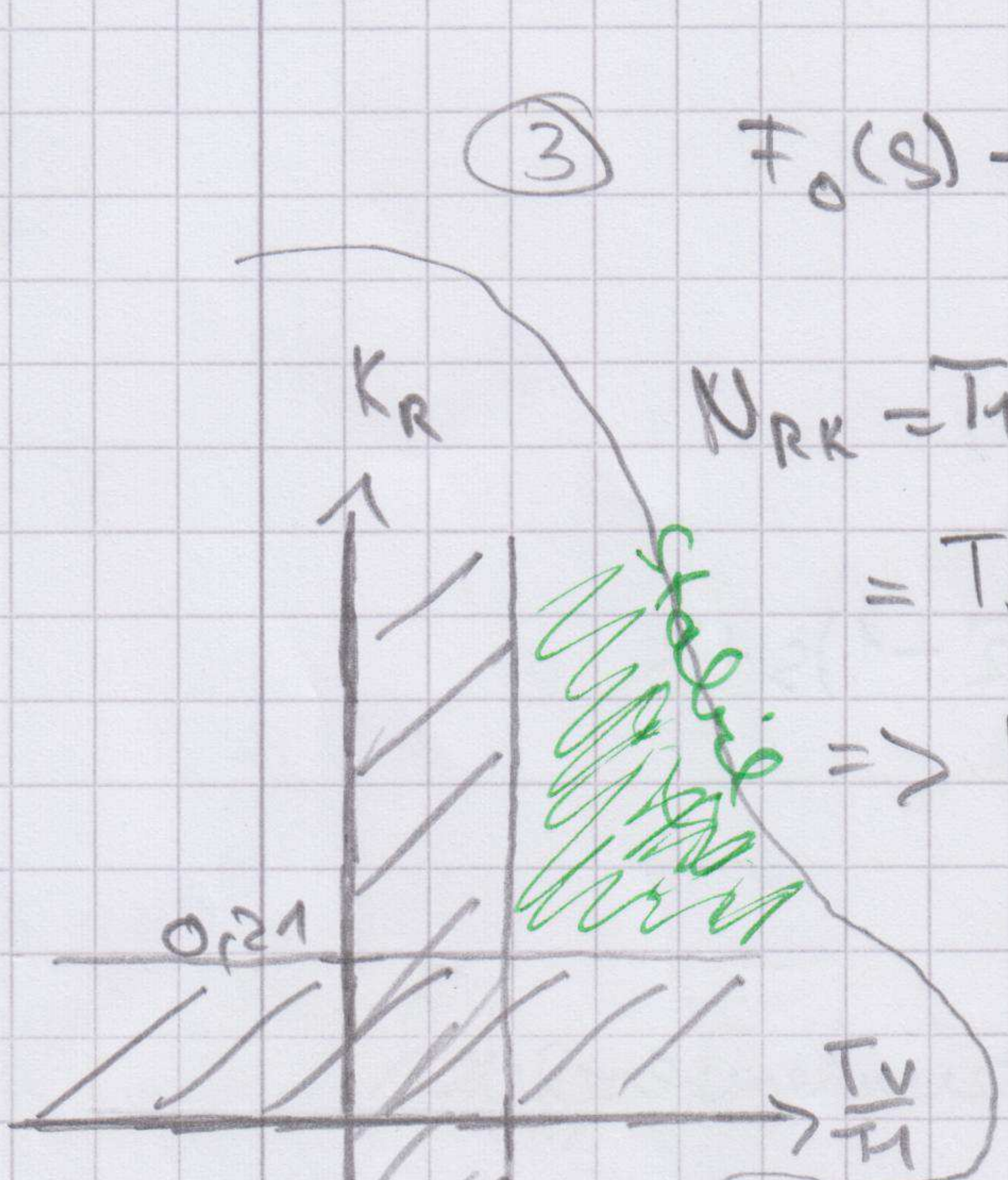
$\Rightarrow p_1 = 6\ ;\ p_2 = -4{,}17$

$F_\infty(0) = \dfrac{F_o}{1+F_o} = \dfrac{\dfrac{2 \cdot 4{,}17 \cdot (1+s)}{s^2 - 4{,}17}}{1 + \dfrac{2 \cdot 4{,}17 \cdot (1+s)}{s^2 - 4{,}17}}$

$= \dfrac{8{,}34(1+s)}{s^2 - 4{,}17 + 8{,}34(1+s)}$

$x_d(\infty) = \lim_{s \to 0} (1 - F_W(s)) = -1$

3 $F_o(s) = K_R \dfrac{1 + T_V \cdot s}{1 + T_1 \cdot s} \cdot \dfrac{19{,}6}{s^2 - 4{,}17}$



$N_{RK} = T_1 s^3 - 4{,}17 T_1 \cdot s + s^2 - 4{,}17 + 19{,}6 K_R T_V \cdot s + 19{,}6 K_R$

$= T_1 s^3 + s^2 + (19{,}6 K_R \cdot T_V - 4{,}17 \cdot T_1) \cdot s + 19{,}6 K_R - 4{,}17$

$\Rightarrow b_0 > 0: 19{,}6 K_R - 4{,}17 > 0 \Rightarrow K_R > \dfrac{4{,}17}{19{,}6} \approx 0{,}21$

$b_3 > 0: T_1 > 0$

$b_1 > 0: \dfrac{4{,}17}{19{,}6} \cdot \dfrac{T_1}{T_V}$

$b_2 > 0: 1 > 0$

$b_1 \cdot b_2 - b_0 b_3 > 0: 19{,}6 \cdot K_R \cdot T_V - 4{,}17 \cdot T_1 - T_1(19{,}6 K_R - 4{,}17) > 0$

$\Rightarrow 19{,}6 T_V - 19{,}6 T_1 > 0 \Rightarrow \dfrac{T_V}{T_0} > 0$