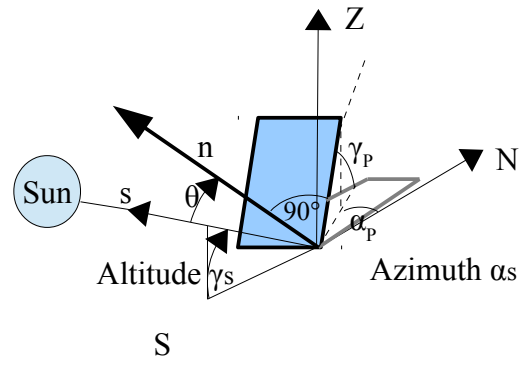
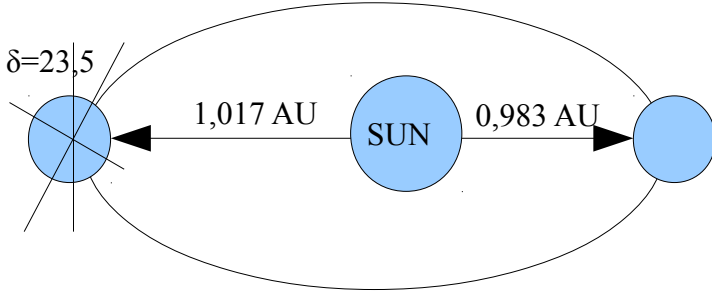


## POSITION OF THE SUN



$$J' = 360^\circ \cdot \frac{J}{365}$$

$$\delta = (0,3948 - 23,2559 \cdot \cos(J' + 9,1^\circ) - 0,3915 \cdot \cos(2 \cdot J' + 5,4^\circ) - 0,1764 \cdot \cos(3 \cdot J' + 105,2^\circ))$$

$$TEQ = (0,0066 + 7,3525 \cdot \cos(J' + 85,9^\circ) + 9,9359 \cdot \cos(2 \cdot J' + 108,9^\circ) + 0,3387 \cdot \cos(3 \cdot J' + 105,2^\circ)) / 60 \cdot h$$

$$\text{Mean Local Time } MLT = (\text{HourOfDay} - TZ + 1h - 4 \cdot (15^\circ - \alpha)) / 60^\circ h$$

$$\text{True Local Time } TLT = MLT + TEQ$$

$$\text{Angle } \omega = (12.00h - TLT) \cdot 15^\circ$$

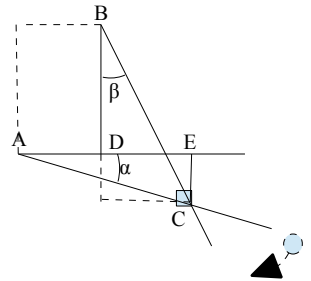
$$\gamma_s = \arcsin(\cos(\omega) \cdot \cos(\varphi) \cdot \cos(\delta) + \sin(\varphi) \cdot \sin(\delta))$$

$$\alpha_s = \begin{cases} -\arccos\left(\frac{\sin(\gamma_s) \cdot \sin(\varphi) - \sin(\delta)}{\cos(\gamma_s) \cdot \cos(\varphi)}\right) & \text{für } TLT \leq 12 \\ +\arccos\left(\frac{\sin(\gamma_s) \cdot \sin(\varphi) - \sin(\delta)}{\cos(\gamma_s) \cdot \cos(\varphi)}\right) & \text{für } TLT > 12 \end{cases}$$

$$\alpha = \arctan\left(\frac{EC}{EA}\right)$$

$$\rightarrow \alpha_s = \alpha + 90^\circ$$

$$\beta = \arctan\left(\frac{DE}{BD + EC}\right)$$



## Power and Energy from the Sun

$$\text{Irradiation Power Density } \dot{G} = \sigma \cdot \epsilon \cdot A \cdot T_s^4$$

$$\text{Irradiation Power Density Sun: } \dot{G}_s = \frac{\dot{G}}{A} = \sigma \cdot \epsilon \cdot T_s^4 = 5,67 \cdot 10^{-8} \frac{W}{m^2 \cdot K^4} \cdot 1 \cdot (5777 K)^4 = 63,15 \frac{MW}{m^2}$$

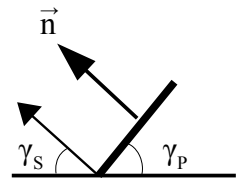
$$\text{Solar Constant: } \dot{G}_0 = \dot{G}_s \cdot \frac{4\pi \cdot R_s^2}{4\pi \cdot (AU)^2} = 63,15 \frac{MW}{m^2} \cdot \frac{(0,6963 \cdot 10^6 km)^2}{(149,6 \cdot 10^6 km)^2} = 1368 \frac{W}{m^2}$$

$$\text{Irradiance } G = \int_T \dot{G} \cdot dt$$

## Transformation of irradiation

$$\vec{n} = \begin{pmatrix} r \cdot \sin(90^\circ - (90^\circ - \gamma_p)) \cdot \cos(\alpha_p) \\ r \cdot \sin(90^\circ - (90^\circ - \gamma_p)) \cdot \sin(\alpha_p) \\ r \cdot \cos(90^\circ - (90^\circ - \gamma_p)) \end{pmatrix} = \begin{pmatrix} \sin(30^\circ) \cos(30^\circ) \\ \sin(30^\circ) \sin(0^\circ) \\ \cos(30^\circ) \end{pmatrix} = \begin{pmatrix} (1/2) \\ 0 \\ (1/2) \cdot \sqrt{3} \end{pmatrix} \wedge |\vec{n}| = 1$$

$$\vec{s} = \begin{pmatrix} r \cdot \sin(90^\circ - \gamma_s) \cdot \cos(\alpha_s) \\ r \cdot \sin(90^\circ - \gamma_s) \cdot \sin(\alpha_s) \\ r \cdot \cos(90^\circ - \gamma_s) \end{pmatrix} \quad \cos(\theta) = \frac{\vec{s} \cdot \vec{n}}{|\vec{s}| \cdot |\vec{n}|} \quad \dot{G}_{Tot} = \dot{G}_{dir} + \dot{G}_{dif} + \dot{G}_{ref}$$



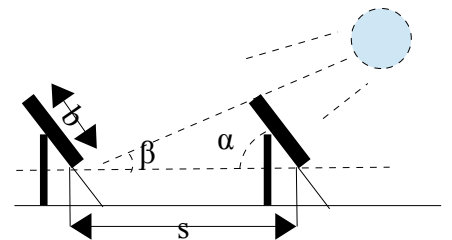
$$\text{Direct Radiation: } \dot{G}_{dir, tilt} = \dot{G}_{dir, hor} \cdot \frac{\cos(\theta)}{\sin(\gamma_s)} \quad \dot{G}_{Tot} = \dot{G}_{dir, tilt} + \dot{G}_{dif, tilt} + \dot{G}_{ref, tilt}$$

$$\text{Diffuse Radiation: } G_{dif, tilt} = \frac{1}{2} \cdot G_{dif, tilt} \cdot (1 + \cos(\gamma_p)) \cdot (1 + F \cdot \sin^3(\frac{\gamma_p}{2})) \cdot (1 + F \cdot \cos^2(\theta) \cdot \cos^3(\gamma_s))$$

$$\text{Cloudness Function: } F = 1 - \left(\frac{G_{dif, hor}}{G_{tot, hor}}\right)$$

$$\text{Ground Reflection: } G_{ref, tilt} = \frac{G_{tot, hor} \cdot A_{Albedo} \cdot 1}{2} \cdot (1 - \cos(\gamma_p))$$

## Shading



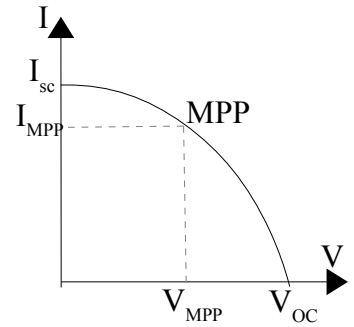
$$s = \frac{b \cdot \sin(180^\circ - (\alpha + \gamma_s))}{\sin(\gamma_s)}$$

# Photovoltaics Basics Modules and Generators

## Photovoltaic Effect

Plank ' s quantum :  $h = 6,62 \cdot 10^{-34} \text{Ws}^2$      $c = 3 \cdot 10^5 \frac{\text{km}}{\text{s}}$      $e = 1,6 \cdot 10^{-19} \text{As}$

Spectral sensitivity :  $S(\lambda) = \frac{e \cdot \lambda}{h \cdot c} \cdot \eta_{ext}$      $E_{ph} = h \cdot f = \frac{h \cdot c}{\lambda} > E_G$



## Operation of Photovoltaic Solar Cells and Modules

Standard Test Conditions : STC  $\rightarrow T_{PV} = 25^\circ$  ;  $AM = 1.5$  ;  $G = \frac{1000 \text{W}}{\text{m}^2}$

Fill Factor :  $FF = \frac{(V_{MPP} \cdot I_{MPP})}{(V_{oc} \cdot I_{sc})}$

Open Circuit voltage :  $V_{oc}(T) = V_{oc}(25^\circ) + \Delta V_{oc} = V_{oc}(25^\circ) + \alpha \cdot V_{oc}(25^\circ) \cdot (T - 25^\circ \text{C})$

Short Circuit Current :  $I_{sc}(T) = I_{sc}(25^\circ) + \Delta I_{sc} = I_{sc}(25^\circ) + \beta \cdot I_{sc}(25^\circ) \cdot (T - 25^\circ \text{C})$

Maximum Power Point :  $P_{MPP}(T) = V_{oc}(25^\circ) + \Delta P_r = P_{MPP}(25^\circ) + \gamma \cdot P_r(25^\circ) \cdot (T - 25^\circ \text{C})$

Photo Current :  $I_{PH}(G) = I_{PH}(STC) \cdot \frac{\dot{G}}{\dot{G}_{STC}} = I_{PH}(1000 \frac{\text{W}}{\text{m}^2}) \cdot \frac{\dot{G}}{1000 \text{Wm}^2}$     Efficiency :  $\eta = \frac{P_{MPP}}{\dot{G} \cdot A_{Module}}$

## Thermal Behavior of the Modules

$\sigma = 5,67 \cdot 10^{-8} \text{W/(m}^2 \cdot \text{K}^4)$      $\epsilon := \text{emis. coef. (Glass : 0,88)}$      $\alpha := \text{Absorp. coef. (Glass : 0,7-0,9)}$

$h_c = (h_{cw}^3 + h_{cfree}^3)^{1/3}$      $h_{cw} := 4,214 + 3575 V_w$      $h_{cfree} := 1,78 (T_{PV} - T_0)^{1/3}$

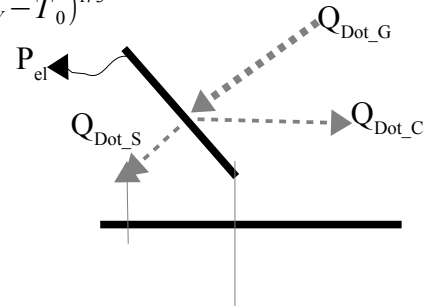
$h_r = \epsilon_{Mod} \cdot \sigma \cdot (T_{PV}^2 + T_0^2) \cdot (T_{PV} + T_0)$

Power fro the Sun :  $\dot{Q} := \alpha_{Mod} \cdot \dot{G} \cdot A_{Mod}$

Long wave irradiation exchange :  $\dot{Q}_s := 2 \cdot A_{Mod} \cdot h_r \cdot (T_{PV} - T_0)$

Convection losses :  $\dot{Q}_c := 2 \cdot [h_{cw}^3 \cdot h_{cfree}^3]^{1/3} \cdot A_{Mod} \cdot (T_{PV} - T_0)$

$P_{el} = \dot{Q}_G - (\dot{Q}_s + \dot{Q}_c)$      $\eta = P_{el} / (\dot{G} \cdot A_{Mod})$

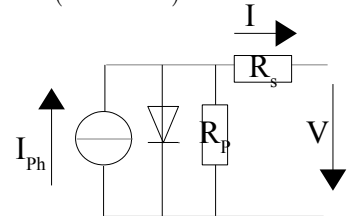


## Modelling: Equivalent Circuits:

$I_s$  (Saturation Curr.) =  $1,852 \cdot 10^{-9} \text{A}$      $k$  (SBC) :=  $1,38046 \cdot 10^{-23} \text{J/K}$      $m$  (Fit Fact.) :=  $1,0..1,2$

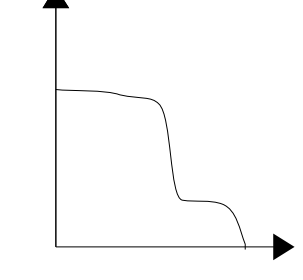
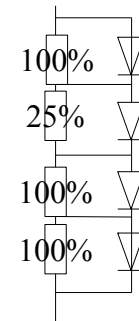
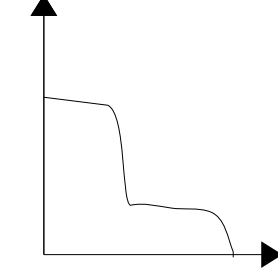
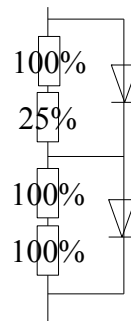
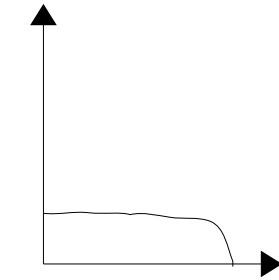
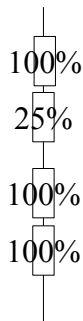
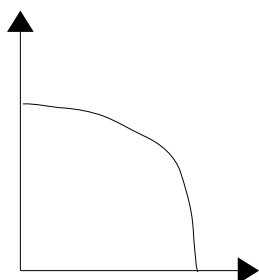
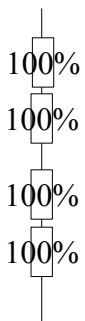
$I_{ph} - I_s \cdot (e^{\frac{V+I \cdot R_s}{m \cdot V_T}} - 1) - \frac{V+I \cdot R_s}{R_p} - I = 0$      $V_T = \frac{k \cdot T}{e} \rightarrow V_T = 0,026 \text{V for } T = (25+273) \text{K}$

For  $n_s$  solar cell :  $I_{ph} - I_s \cdot (e^{\frac{V+I \cdot R_s}{n_s \cdot m \cdot V_T}} - 1) - \frac{V+I \cdot R_s}{n_s \cdot R_p} - I = 0 = f(I) = 0$



## Shading

$I_{Ph} = \frac{I_{sc} \cdot \dot{G}}{1000 \text{W/m}^2} = I_{sc} \cdot a$  ( $a := \text{of Irr. Received}$ )





## Grid Connection Photovoltaic Systems

### Efficiency

European Efficiency:  $\eta = 0,03 \cdot \eta_5 + 0,06 \cdot \eta_{10} + 0,13 \cdot \eta_{20} + 0,1 \cdot \eta_{30} + 0,48 \cdot \eta_{50} + 0,2 \cdot \eta_{100}$

#### Master Slave Operation or Team Operation

P/kW	0	12,5	25	50	75	100	125	150	200	250
P/P <sub>rOneUnit</sub>	0%	5%	10%	20%	30%	40%	50%	60%	80%	100%
$\eta_{OneUnit}$	0,0%	58,0%	78,0%	90,5%	93,6%	94,1%	94,4%	94,8%	95,5%	94,6%

#### From the load factor 50% on both inverters take over the same load

P/kW	0	25	50	100	150	200	250	300	400	500
P/P <sub>rOneUnit</sub>	0%	5%	10%	20%	30%	40%	50%	60%	80%	100%
$\eta_{OneUnit}$	0,0%	78,0%	90,5%	94,1%	94,8%	95,5%	94,4%	94,8%	95,5%	94,6%

$$\text{Effici. Of PV-Gen} : \delta_{PV-Gen} := \frac{W_{DC}}{G_{Irr.} \cdot A_{Area}} \quad \eta_{INV} := \frac{W_{AC}}{W_{DC}} \quad G_{Modul} = G_{Hor.} \cdot \text{Tiltfaktor}$$

$$\text{Rated Power } P_r = G_{STC} \cdot A \cdot \eta_{STC} \quad \eta_{Mod, STC} = \frac{P_r}{G_{STC} \cdot A_{Mod}}$$

$$\text{Final Yield} : Y_F := \frac{W_{AC}(\text{Grid})}{P_r} = \frac{G_{Modul} = G_{Hor.} \cdot \text{Tiltfaktor} \cdot A \cdot \eta_{WR} = \text{MeanEffInv} \cdot \eta_{PV-notSTC}}{P_r}$$

$$\text{Performance Ratio} : PR := \frac{Y_F \cdot G_{STC}}{G} = \frac{W_{AC}}{G \cdot A \cdot \eta_{STC}} = \frac{Y_F \cdot P_r}{G \cdot A \cdot \eta_{STC}}$$

#### W<sub>AC</sub> of a Location X in a Year with n in Serial and m Parallel PV Modules:

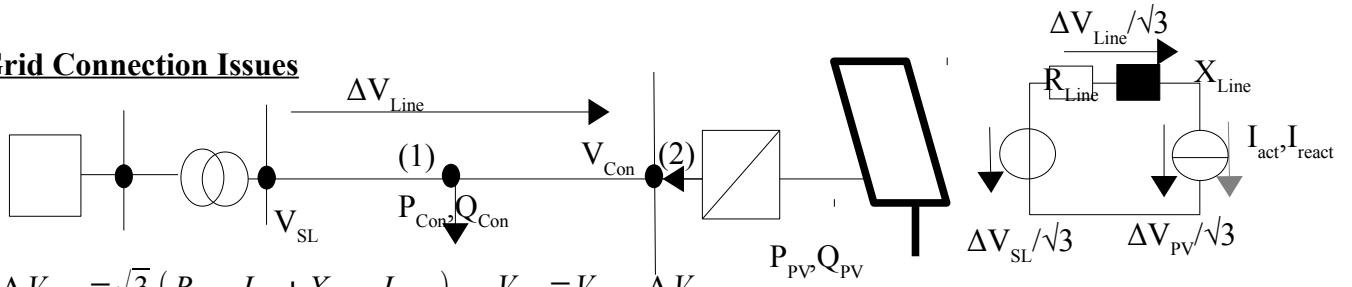
$$W_{LocationX} = G_{LocationX} \cdot n_{ser} \cdot m_{par} \cdot A_{Mod} \cdot \eta_{Mod-not-STC} \cdot \eta_{Inv} \cdot \eta_{Cab}$$

#### Wiring and Cable Losses of a Grid connected PV System

$$n_{String} = \frac{\text{Modules}_{Total}}{\text{Modules}_{perString}} \quad \Delta U_{Lost1Phase} = 2 \cdot I \cdot R = 2 \cdot I \cdot \frac{l_{cable}}{\kappa_{cable} \cdot A_{cable}} \quad \kappa_{Cu} = 56 \frac{S \cdot m}{mm^2}; \kappa_{Al} = 35 \frac{S \cdot m}{mm^2}$$

$$P_{Looses-1Ph-ACorDC} = 2 \cdot I^2 \cdot R = 2 \cdot I \cdot \frac{l_{cable}}{\kappa_{cable} \cdot A_{cable}} \quad P_{Looses-3Ph-ACorDC} = 3 \cdot I^2 \cdot R = 2 \cdot I \cdot \frac{l_{cable}}{\kappa_{cable} \cdot A_{cable}}$$

## Grid Connection Issues



$$\Delta V_{Line} = \sqrt{3} \cdot (R_{Line} \cdot I_{act} + X_{Line} \cdot I_{react}) \quad V_{PV} = V_{SL} - \Delta V_{Line}$$

$$I_{actCon} = \frac{P_{Con}}{\sqrt{3} \cdot V_N} \quad I_{actPV} = \frac{-P_{PV}}{\sqrt{3} \cdot V_N} \quad R_{SL-2} = l_{SL-2} \cdot \Omega / km$$

$$R_{1-2} = l_{1-2} \cdot \Omega / km \quad I_{react} = I_{act} \cdot \tan(\arccos(\cos(\varphi))) = \frac{Q}{\sqrt{3} \cdot V_N} \quad \varphi = \frac{P}{\sqrt{P^2 + Q^2}}$$

$$\Delta V_{SL-1} = \sqrt{3} (R_{SL-1} \cdot (I_{actCon} + I_{actPV}) + X_{SL-1} \cdot (I_{reactCon} + I_{reactPV}))$$

$$\Delta V_{1-2} = \sqrt{3} (R_{1-2} \cdot (I_{PVact}) + X_{1-2} \cdot (I_{PVreact})) \quad V_1 = V_{SL} - \Delta V_{SL-1} \quad V_2 = V_1 - \Delta V_{1-2}$$

## Characteristics of Batteries

$Q_{in}$  = Charge in Ah into the battery     $Q_{Batt}$  = Charge in Ah in the battery

$Q_{out}$  = Charge in Ah in the battery

$$DOD = \frac{Q_{out}}{Q_{in}} \quad SOC = \frac{Q_{Batt}}{Q_{in}} = 1 - DOD \quad W_{BattTotal} = \frac{W_{AC}}{\eta_{DC-AC} \cdot \eta_{BatOut} \cdot \eta_{BatDOD}}$$

## Battery SOC from t1 to t2

$$Capacity_{Batt} = \frac{W_{BattTotal}}{Volt_{Batt}} \quad W_{PV\ t1-t2} = G_{t1-t2} \cdot A_{totalPVArea} \cdot \Delta t \cdot \eta_{PV} \cdot \eta_{DC-DC} \quad W_{Load\ t1-t2} = \frac{\sum Load_h \cdot 1h}{\eta_{DCtoAC}}$$

$$W_{\Delta} = W_{PV\ t1-t2} - W_{Load\ t1-t2} \quad \text{For } W_{\Delta} < 0: W_{Batt2} = W_{Batt1} - \frac{W_{\Delta}}{\eta_{BatOut}}$$

$$\text{For } W_{\Delta} > 0: W_{Batt2} = W_{Batt1} + W_{\Delta} \cdot \eta_{BatIn} \quad SOC_{t2} = \frac{W_{Batt2}}{W_{BattTotal}}$$

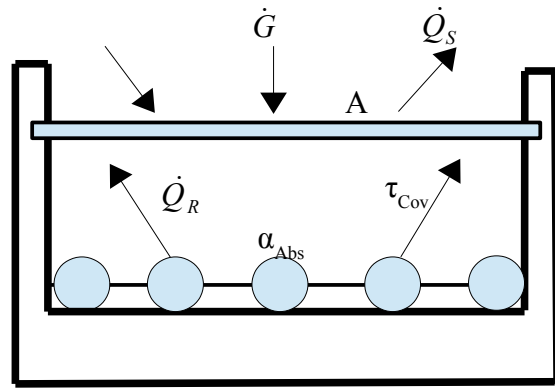
# SOLAR THERMAL SYSTEM

Absorbtion coeff. =  $\alpha$ ; Reflection factor =  $\rho$ ; Transmission coef. =  $\tau$

$$\alpha(\lambda) = \frac{\text{Absorbed radiation}}{\text{Total radiation}} \leq 1 \quad \alpha(\lambda) + \rho(\lambda) + \tau(\lambda) = 1 \quad \text{Because of isolation } \tau = 0 \rightarrow \alpha = 1 - \rho$$

## Power Output and Efficiency

$\tau$	=Transmission coeff. Of cover
$\alpha$	=Absor. coeff. of absorber
$\dot{G}$	=Power of Irradiation
$\dot{Q}_N = \dot{Q}_{use}$	=Useful power transferred to heat fluid
$\dot{Q}_S$	=Reflection losses of the glass cover
$\dot{Q}_R$	=Reflection losses of the absorber
$\dot{Q}_{Loss}$	=Power emitted in the long wave area + losses due to convection
$T_{Abs.}$	=Temperature of absorber
$T_{Amb}$	=Ambient temperature
$a_0, a_1, a_2$	=Given parameters of collector
$\alpha_{Abs.} \cdot \tau_{Cov} = a_0$	=Optic losses



Construction of a collector

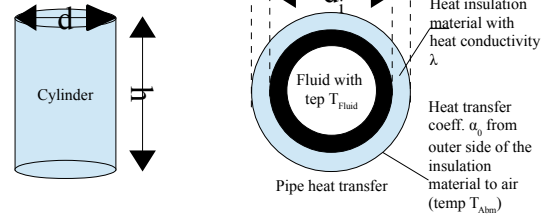
$$\dot{Q}_{Use} = (\dot{G} \cdot A \cdot \tau_{Abs.}) - a_1 \cdot A \cdot (T_{Abs.} - T_{Amb}) - a_2 \cdot A \cdot (T_{Abs.} - T_{Amb})^2$$

$$\eta_{Coll} = \frac{\dot{Q}_N}{(\dot{G} \cdot A \cdot \tau_{Abs.})} = \alpha_{Abs.} \cdot \tau_{cov} - \frac{a_1}{\dot{G}} \cdot (T_{Abs.} - T_{Amb}) - \frac{a_2}{\dot{G}} \cdot (T_{Abs.} - T_{Amb})^2$$

$$\eta_{Coll} = \frac{\dot{Q}_N}{(\dot{G} \cdot A \cdot \tau_{Abs.})} = a_0 - \frac{a_1}{\dot{G}} \cdot (T_{Abs.} - T_{Amb}) - \frac{a_2}{\dot{G}} \cdot (T_{Abs.} - T_{Amb})^2$$

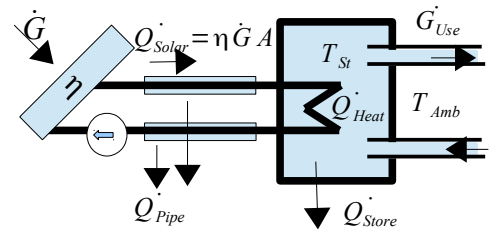
## Thermal Stores:

m	Mass of water in store	$T_{St}$	Temperature of water in store
c	4,1868 kJ / (kg*K) = specific heat of water	$T_{Amb}$	Ambient temperature
h	Height of Cylinder	d	Diameter of Cylinder
$U_{St}$	Heat transfer coefficient	$A_{St}$	Surface area
l	Length of the Pipe		



$$Q = c \cdot m \cdot (T_{St} - T_{Amb.}) \quad A_{St} = 2 \cdot \left(\frac{d}{2}\right) \cdot \pi \cdot h + 1 \cdot \left(\frac{d^2 \cdot \pi}{4}\right) \quad Q_{Loss} = U_{St} \cdot A_{St} \cdot (T_{St} - T_{Amb.})$$

$$U_{Pipe} = \frac{\pi}{\frac{1}{2 \cdot \lambda} \cdot \ln\left(\frac{d_0}{d_i}\right) + \frac{1}{\alpha_o \cdot d_0}} \quad \dot{Q} = l_{Pipe} \cdot U_{Pipe} \cdot (T_{Fluid} - T_{Amb})$$



## Operational behaviour

$$\underbrace{\dot{G} \cdot A \cdot \eta}_{\dot{G}_{Solar}} - \dot{Q}_{pipe} - \dot{Q}_{Use} - \dot{Q}_{Loss} = \frac{dQ_{Heat}}{dt} = \frac{c_p \cdot m \cdot dT_{St}}{dt} = c_p \cdot m \cdot \frac{\Delta T_{St}}{\Delta t}$$

$$\Delta T_{St} = \Delta \frac{t}{c_p \cdot m} \cdot (\dot{G} \cdot A \cdot \eta - \dot{Q}_{Pipe} - \dot{Q}_{Store} - \dot{Q}_{Use})$$

$$\text{Solar Fraction: } SF = \frac{Q_{offer} (Solar)}{Q_{demand}}$$

## Rough design of solar thermal systems

Q	Required heat energy	c	4,19kJ/(kg·K)
m	Mass of warm water of temperature T <sub>2</sub> (m=V·ρ)	ΔT	Temperature of warm water T <sub>2</sub> minus temperature of cold water T <sub>1</sub> surface of absorber
A	Surface of absorber	δ	Mean efficiency
G	Mean irradiation per day	V	Volumen of required Water
ρ	Density of Water 1kg/l		

$$Q_{requiredHeat} = m \cdot c \cdot \Delta T = V \cdot \rho \cdot c \cdot (T_{Hot} - T_{Cold}) \quad A_{requiredArea} = \frac{Q}{G \cdot \delta}$$

### Concentration Reflecting Trough Collectors

A <sub>R</sub>	Reflector surface = aperture	F	Absorber surface = A <sub>R</sub> /C
R <sub>S</sub>	Distance Sun Earth = 0,695·10 <sup>8</sup> km	σ	Temperature of Absorber Stefan Boltzmann law. 5,67·10 <sup>-8</sup> W·m <sup>-2</sup> ·K <sup>-4</sup>
C	Concentration Factor C <sub>max</sub> =46211	T <sub>S</sub>	Temperature of Sun = 5777K
G <sub>dot,s</sub>	Direct radiation	T <sub>Abs</sub>	Temperature of Absorber
G <sub>dot,0</sub>	Radition with reaches earths surface	ρ <sub>Abs</sub>	Reflection coeff. Of reflector =(1/ρ <sub>R</sub> )
ρ <sub>R</sub>	Reflection coeff. Of reflector	ρ <sub>Abs</sub>	Reflection coeff. Of absorber=(1/ρ <sub>R</sub> )
U <sub>Abs</sub>	Heat transmission coeff. Of absorber in W/(m <sup>2</sup> ·K)	α <sub>R</sub>	Emission coeff of reflector =(1-ρ <sub>R</sub> )
T <sub>Amb</sub>	Ambient temperature	Q <sub>dot,N</sub>	Useful Power
m <sub>Dot</sub>	Mass Flow	c <sub>p</sub>	Heat capacity

$$Concentration\ Factor : C = \frac{A_{Abs.}}{F} \quad \frac{1}{C_{max}} = \frac{R_s^2}{(AE)^2} \quad \dot{G}_s = \sigma \cdot T_s^4 \quad \dot{G}_0 = \frac{\dot{G}_s \cdot 4 \cdot \pi \cdot R_s^2}{4 \cdot \pi \cdot (AE)^2}$$

$$\dot{G}_0 \cdot A_R = \sigma \cdot T_s^4 \cdot A_R \cdot \frac{R_s^2}{(AE)^2} \quad \dot{G}_0 \cdot A_R = F \cdot \sigma \cdot T_{Abs}^4$$

$$T_{Abs}^4 = T_s^4 \cdot \frac{R_s^2 \cdot A_R}{(AE)^2 \cdot F} = T_s^4 \cdot \left(\frac{C}{C_{max}}\right) \rightarrow T_{Abs} = T_s \cdot \sqrt[4]{\frac{C}{C_{max}}}$$

$$Energy\ on\ aperture\ coming\ f\ from\ the\ sun = \dot{G} \cdot A_R \quad Energy\ absorption\ on\ reflector = \alpha_R \cdot G \cdot A_R$$

$$Energy\ reflection\ on\ absorber\ as\ F = A_R / C := \rho_R \cdot \rho_A \cdot \dot{G} \cdot F = \rho_R \cdot \rho_A \cdot \dot{G} \cdot \frac{A_R}{C}$$

$$Energy\ convection\ losses\ on\ absorber := U_A \cdot F \cdot (T_{Abs.} - T_{Amb}) = U_A \cdot \frac{A_R}{C} \cdot (T_{Abs.} - T_{Amb})$$

$$Emitted\ energy\ by\ absorber := \epsilon_{Abs.} \cdot F \cdot \sigma \cdot (T_{Abs.}^4 - T_{Amb}^4) = \epsilon_{Abs.} \cdot \frac{A_R}{C} \cdot \sigma \cdot (T_{Abs.}^4 - T_{Amb}^4)$$

$$\dot{Q}_N = \dot{G} \cdot A_R \cdot [\rho_R(1 - \rho_{Abs.}) - \frac{U_A}{\dot{G} \cdot C} \cdot (T_{Abs.} - T_{Amb}) - \epsilon_{Abs.} \cdot \sigma]$$

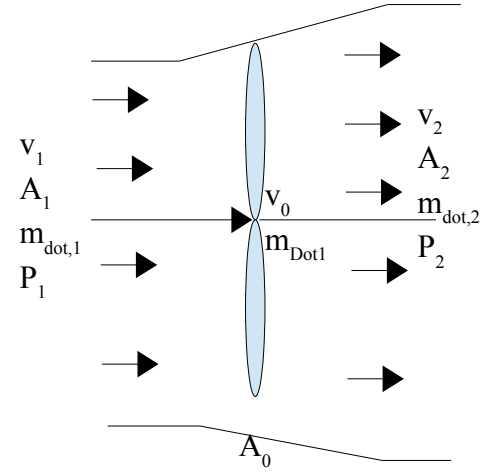
$$\eta = \frac{\dot{Q}_N}{\dot{G} \cdot A_R} = \rho_R(1 - \rho_{Abs.}) - \frac{U_A}{\dot{G} \cdot C} \cdot (T_{Abs.} - T_{Amb}) - \frac{\epsilon_{Abs.} \cdot \sigma}{\dot{G} \cdot C} \cdot (T_{Abs.}^4 - T_{Amb}^4)$$

$$Mass\ flow : \dot{m} = \frac{\dot{Q}_{Abs.}}{c_{POil} \cdot \Delta T}$$



# Wind Energy Converters WEC

a	Hellman-exponent	$V_H$	Velocity of Wind in 10 height
A	Scaling factor	C	Sharpe parameter
$v_1$	Wind speed before Rotor	$V_m$	Medium wind speed
$c_p$	Power coeff.	$\lambda$	Speed coefficient
$\rho$	Density of Air	p	Power per m <sup>2</sup>
$P_{Wind}$	Power from Wind	$A_0$	Rotor Circle Area
$P_{sh}$	Shaft power	$P_{el}$	Power fed to the Grid



## Dependence of the wind speed on the height

$$v_H = v_{10} \cdot \left(\frac{H}{10 \cdot m}\right)^a \quad a = \frac{\ln\left(\frac{v_H}{v_{10}}\right)}{\ln\left(\frac{H}{10 \cdot m}\right)}$$

Conversion of wind speed  $v_1$  of height  $h_1$  to  $v_2$  of height  $h_2$

$$v_2(h_2) = v_1(h_1) \cdot \frac{\ln\left(\frac{h_2}{Z_0}\right)}{\ln\left(\frac{h_1}{Z_0}\right)}$$

## Idealised wind speed distribution

Weibull distribution

$$f(v) = \frac{C}{A} \cdot \left(\frac{v}{A}\right)^{(C-1)} \cdot e^{-\left(\frac{v}{A}\right)^C} \quad v_m = A \cdot \left(0,568 + \frac{0,434}{C}\right)^{\left(\frac{1}{C}\right)}$$

For  $C=2$  and  $A=(v_m \cdot 2) / \sqrt{\pi}$  the Willbull-distribution goes over into the Rayleigh-distribution

$$f(v) = \frac{\pi}{2} \cdot \left(\frac{v}{v_m}\right) \cdot e^{-\left(\frac{\pi}{4}\right) \cdot \left(\frac{v}{v_m}\right)^2}$$

Example:  $v_m = 7,0$  m/s ;  $t = 8760$  h ;  $v = 5,0$  m/s ; class of wind 4,95-5,05 m/s ; t in a year for this class of wind ?

$$f\left(5 \cdot \frac{m}{s}\right) = \frac{\pi}{2} \cdot \frac{5 \cdot m \cdot s^2}{7^2 \cdot s \cdot m^2} \cdot e^{-\left(\frac{\pi}{4}\right) \cdot \left(\frac{5}{7}\right)^2} = 0,1074 \cdot \frac{s}{m}$$

$$T(4,95 - 5,05 \frac{m}{s}) = 0,1074 \cdot \frac{s}{m} \cdot 8760 h \cdot (|4,95 - 5,05| \frac{m}{s}) = 94 h$$

Offered wind power

$$1 N \cdot m = 1 W \cdot s = 1 \frac{kg \cdot m^2}{s} \cdot 1 \frac{W}{m^2} = 1 \frac{kg}{s^3} \quad v_0 = \frac{1}{2} \cdot (v_1 + v_2) \quad \dot{m}_0 = \dot{m}_1 = \dot{m}_2$$

$$P_{Wind} = \frac{1}{2} \cdot \rho \cdot A_0 \cdot v_0^3 \quad p = \frac{P}{A}; [p] = \frac{W}{m^2}$$

Power coefficient  $c_p$  und speed coefficient  $\lambda$  :  $r$  = Rotor radius;  $n$  = Revolutions per second [n]=Hz;  $v$  = wind speed at hub height in front of WEC

$$c_p = \frac{P_1 - P_2}{P_{Wind}} = \frac{1}{2} \cdot \left(1 + \frac{v_2}{v_1}\right) \cdot \left(1 - \left(\frac{v_2}{v_1}\right)^2\right) = \frac{P_r}{\frac{1}{2} \cdot \rho \cdot A_{WEC} \cdot \eta_{mech} \cdot v_r^3} \quad \lambda = \frac{2 \cdot \pi \cdot r \cdot n}{v} \quad P_{sh} = P_{wind} \cdot c_p$$

Output electric power of a WEC:  $P_{el}$

$$P_{el} = \frac{1}{2} \cdot \rho \cdot A \cdot v^3 \cdot \eta = \frac{1}{2} \cdot \rho \cdot \frac{d^2 \cdot \pi}{4} \cdot c_p \cdot \eta \quad v_{rated \text{ wind speed}} = \sqrt[3]{\frac{P_{el}}{\frac{1}{2} \cdot \rho \cdot A \cdot c_p \cdot \eta}}$$

Output energy

Example: A class of Wind speed 9,5 – 10,5 m/s;  $\Delta v = 1$  m/s;  $C = 2,13$  ;  $A = 8,0$  m/s ( $A =$  skaling Faktor not Area)

$$f(v) = \frac{2,3 \cdot s}{8 \cdot m} \cdot \left(\frac{10 \cdot m/s}{8 \cdot m/s}\right)^{(C-1)} \cdot e^{-\left(\frac{10}{8}\right)^{2,13}} = 0,06859 \cdot \frac{s}{m} \quad T_{Class} = 0,06859 \frac{s}{m} \cdot 8760 h \cdot 1 \frac{m}{s} = 600,8 h$$

$$W_{Class} = P_{Class} \cdot T_{Class}$$