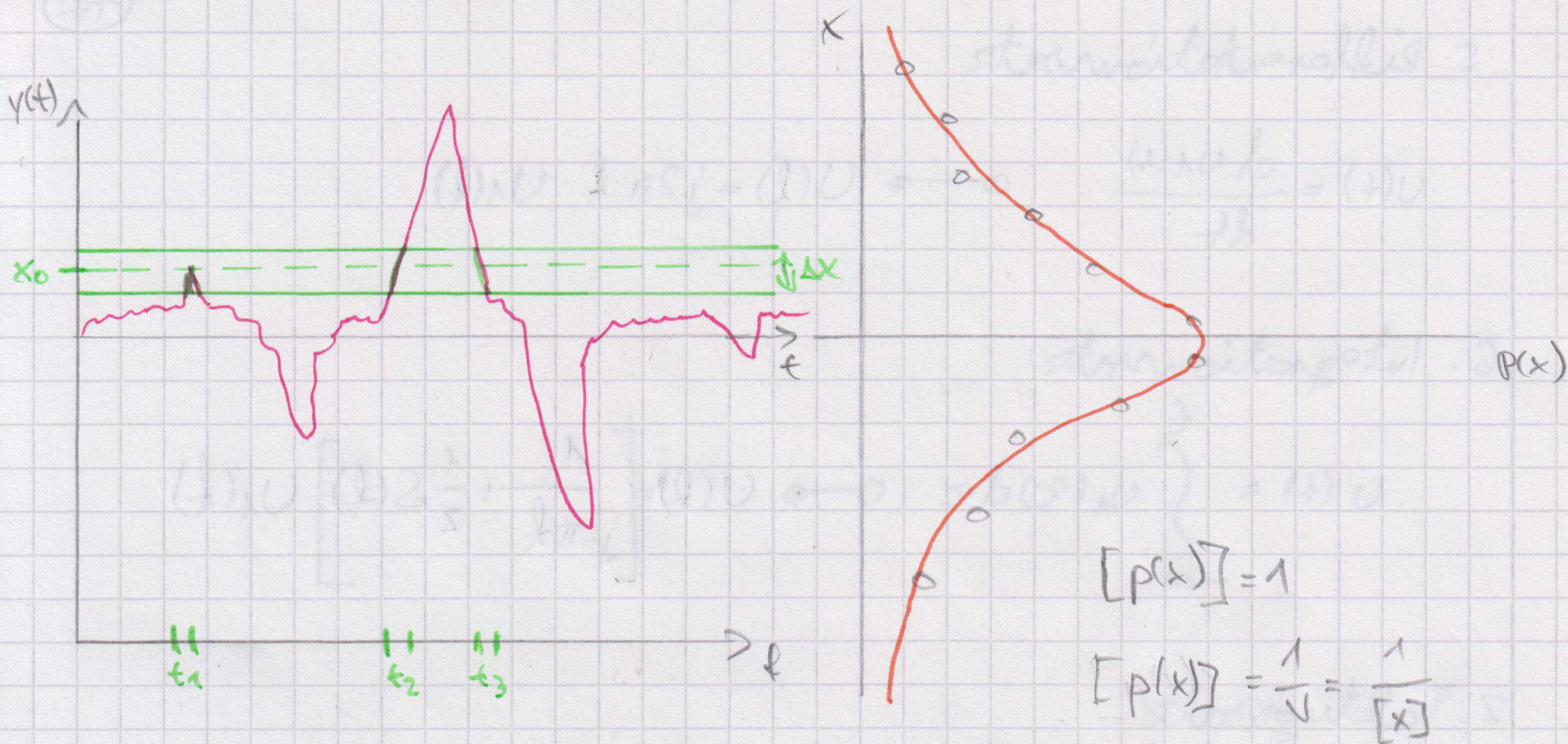




ges. Wahrscheinlichkeit für die Amplituden

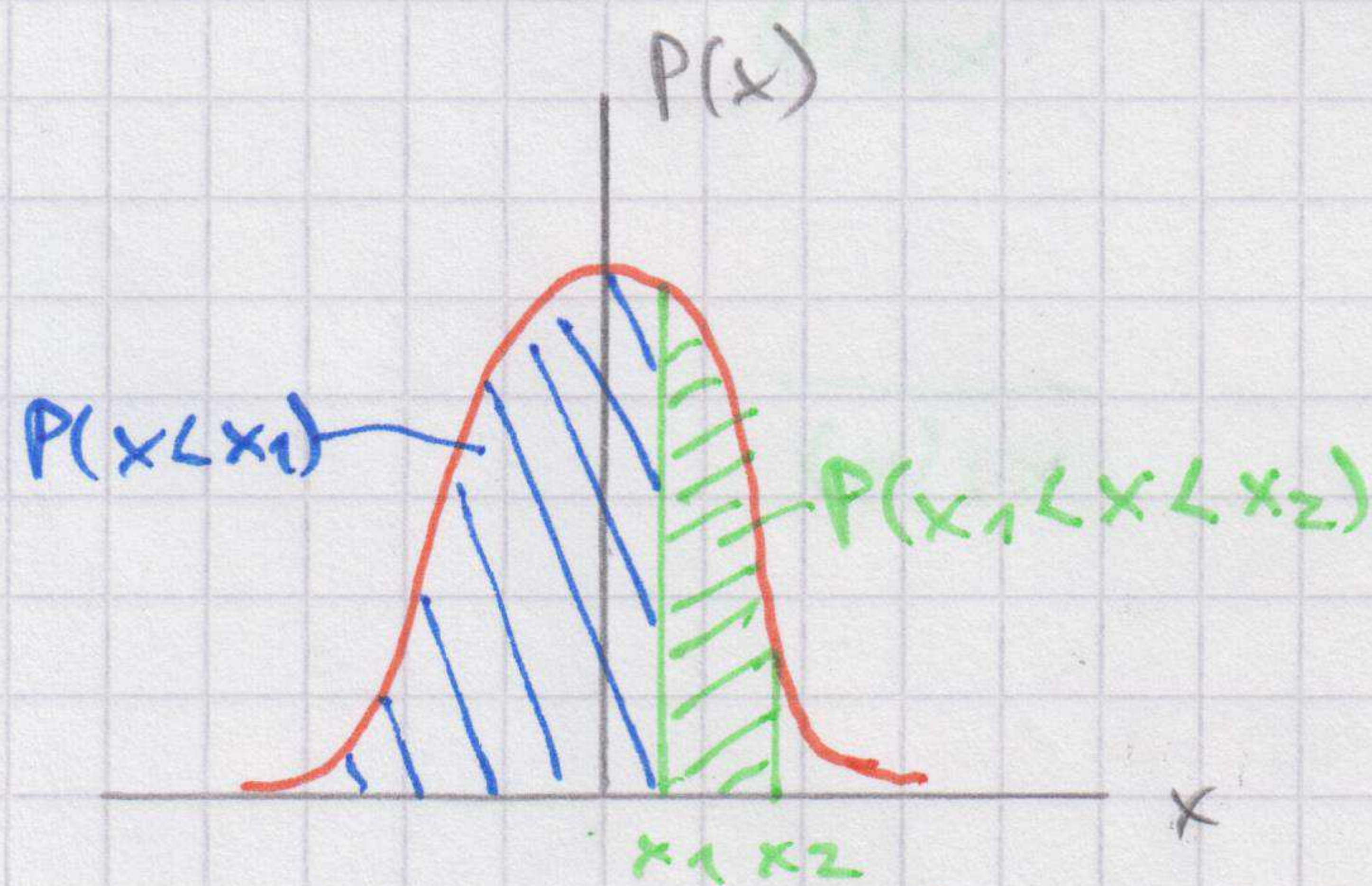
$$P\left(x_0 - \frac{\Delta x}{2} < x(t) < x_0 + \frac{\Delta x}{2}\right) = \frac{t_1 + t_2 + t_3 + \dots}{\text{Gesamtzeit} (\rightarrow \infty)}$$

Grenzübergang für $\Delta x \rightarrow 0$

$P(x) \rightarrow 0$

↳ Wahrscheinlichkeitsdichte

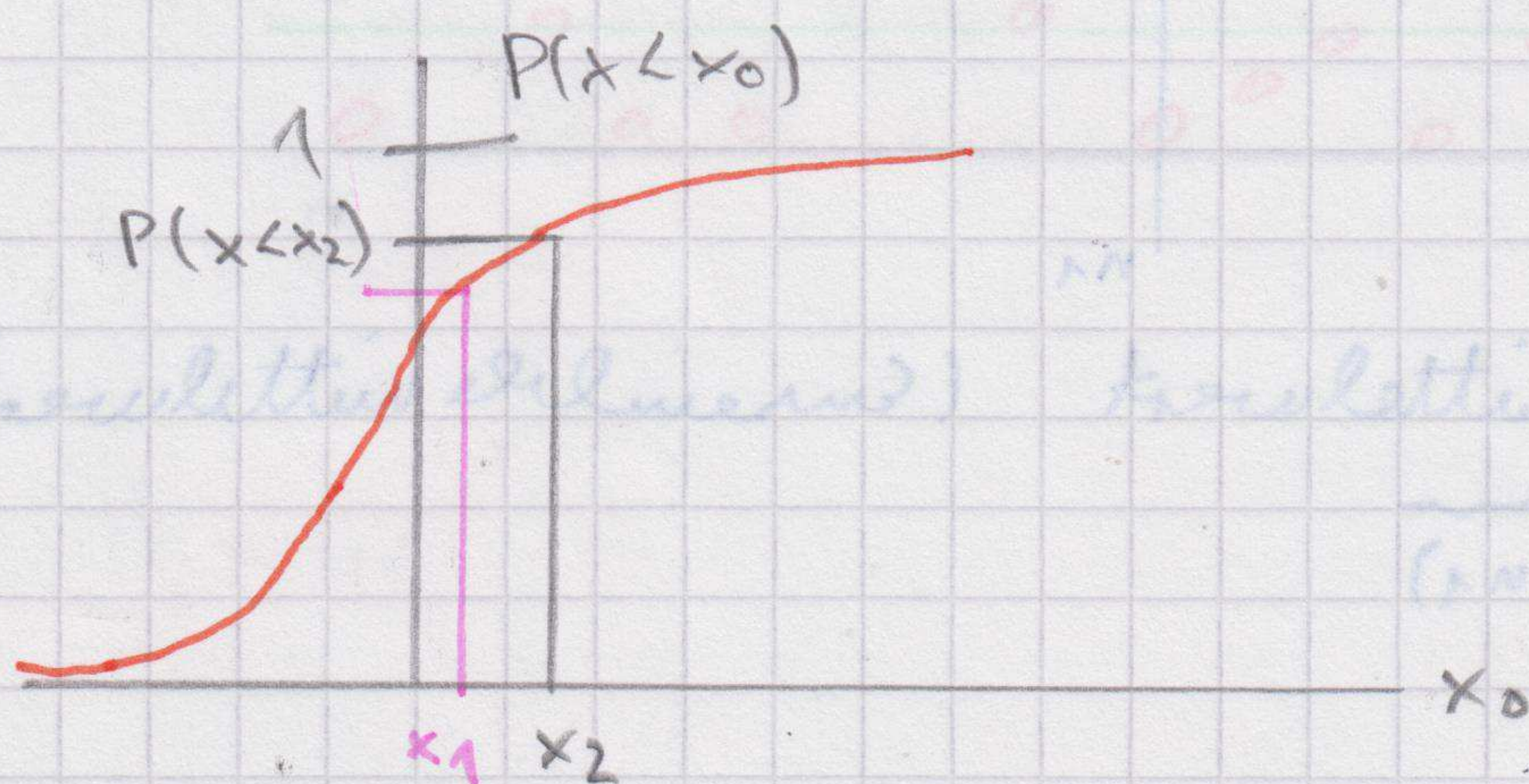
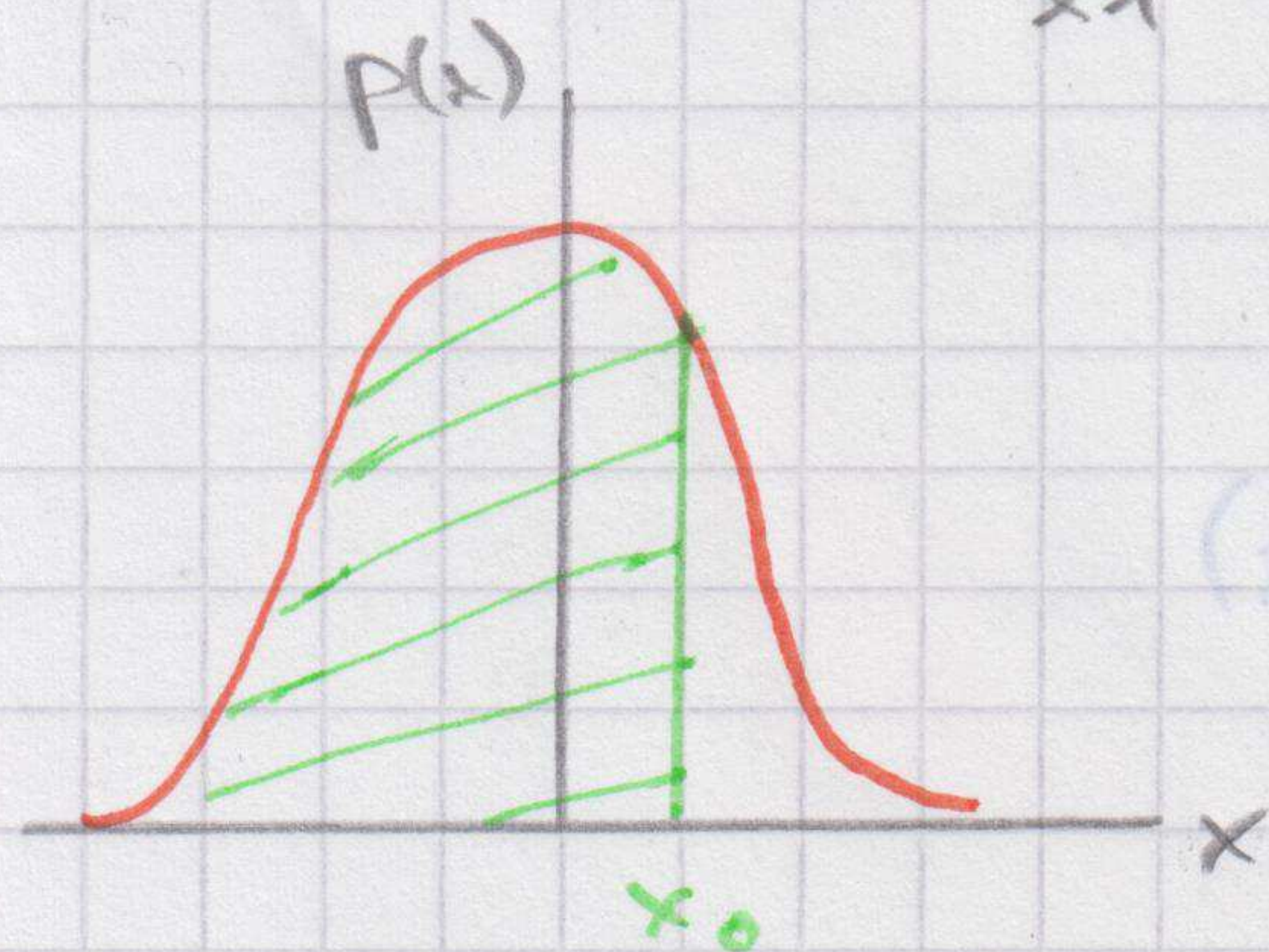
$$p(x) = \lim_{\Delta x \rightarrow 0} \frac{P\left(x_0 - \frac{\Delta x}{2} < x < x_0 + \frac{\Delta x}{2}\right)}{\Delta x}$$



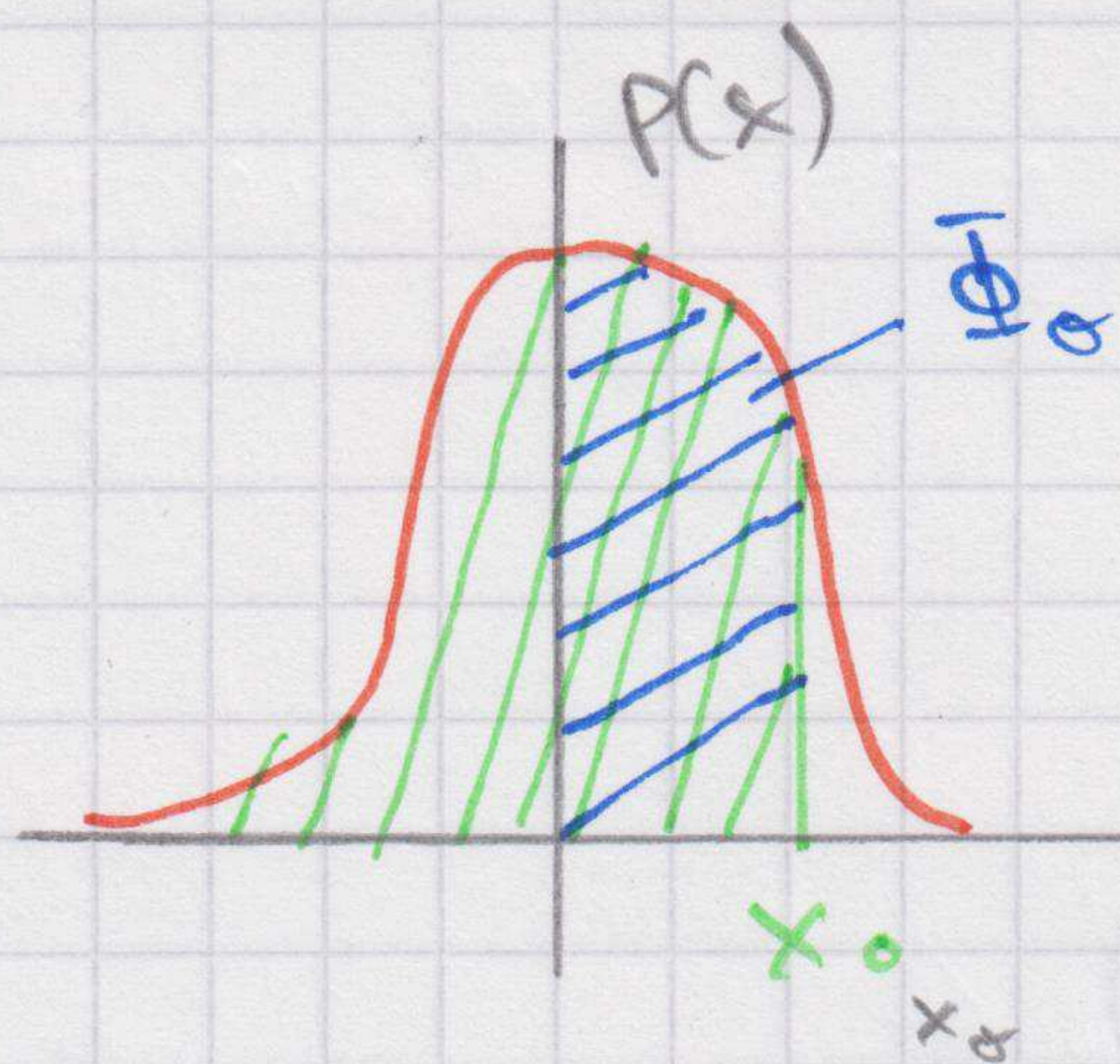
$$\sum_{i=1}^{\infty} p(x_i) = 1$$

$$\int_{-\infty}^{\infty} p(x) dx = 1$$

$$P(x_1 < x < x_2) = \int_{x_1}^{x_2} p(x) dx = p(x < x_2) - p(x < x_1)$$



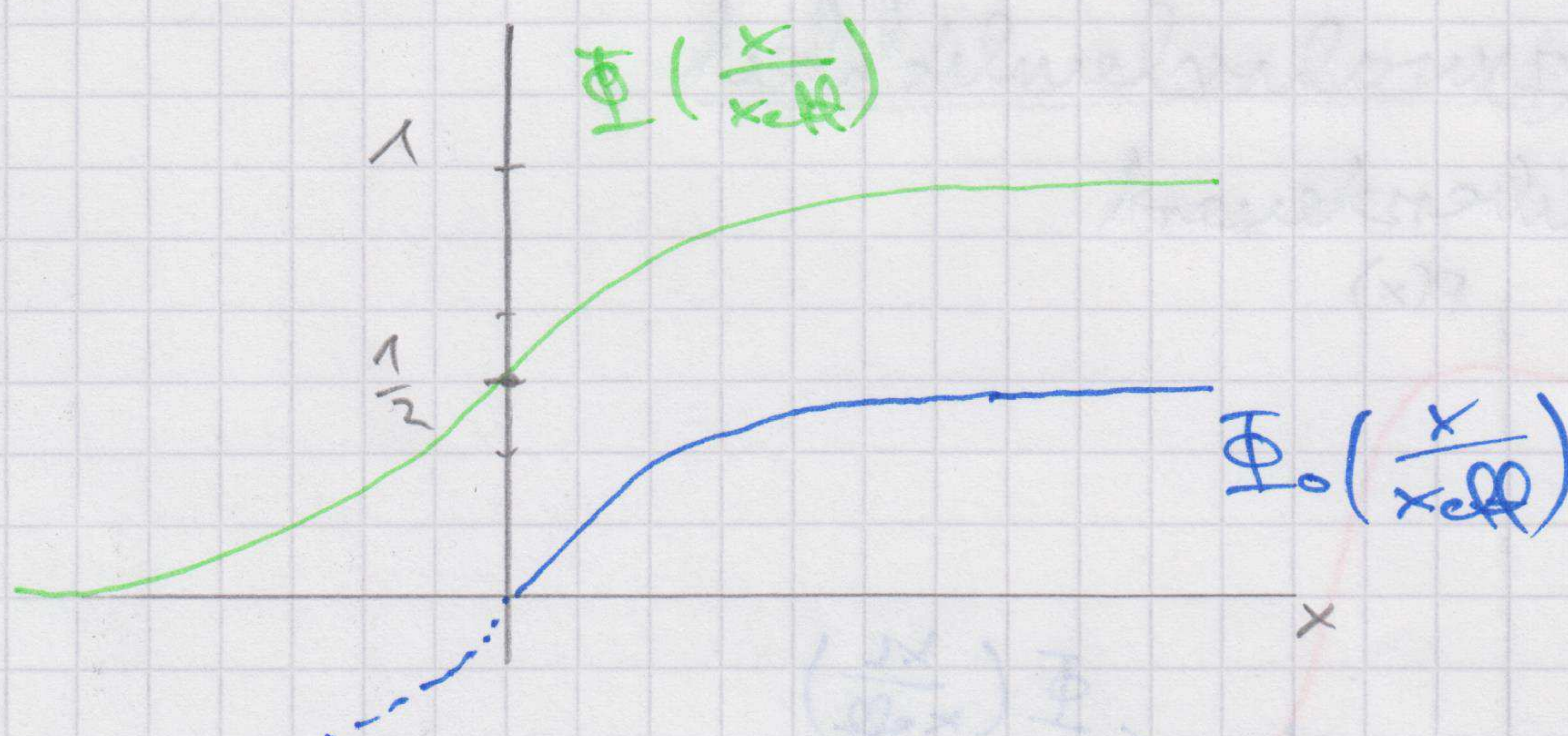
$$P(x < x_0) = \int_{-\infty}^{x_0} p(x) dx$$



Zentrierte Gaußverteilung
($\mu = x_0 = \sigma$)

$$P(x < x_0) = \int_{-\infty}^{x_0} \frac{1}{\sqrt{2\pi} x_{eff}} e^{-\frac{x^2}{2x_{eff}^2}} dx = \int_{-\infty}^{x_0} P(x) dx$$

$$= \Phi\left(\frac{x_0}{x_{eff}}\right) \quad \text{"Gauß'sches Fehlerintegral"}$$



Einseitiges Gauß'sches Fehlerintegral

$$\Phi_0\left(\frac{x_0}{x_{eff}}\right) = \Phi\left(\frac{x}{x_{eff}}\right) - \frac{1}{2} = \int_0^{x_0} p(x) dx$$

Casus-Rechner:

$$P(x) = \Phi(x)$$

$$Q(x) = |\Phi_0(x)|$$

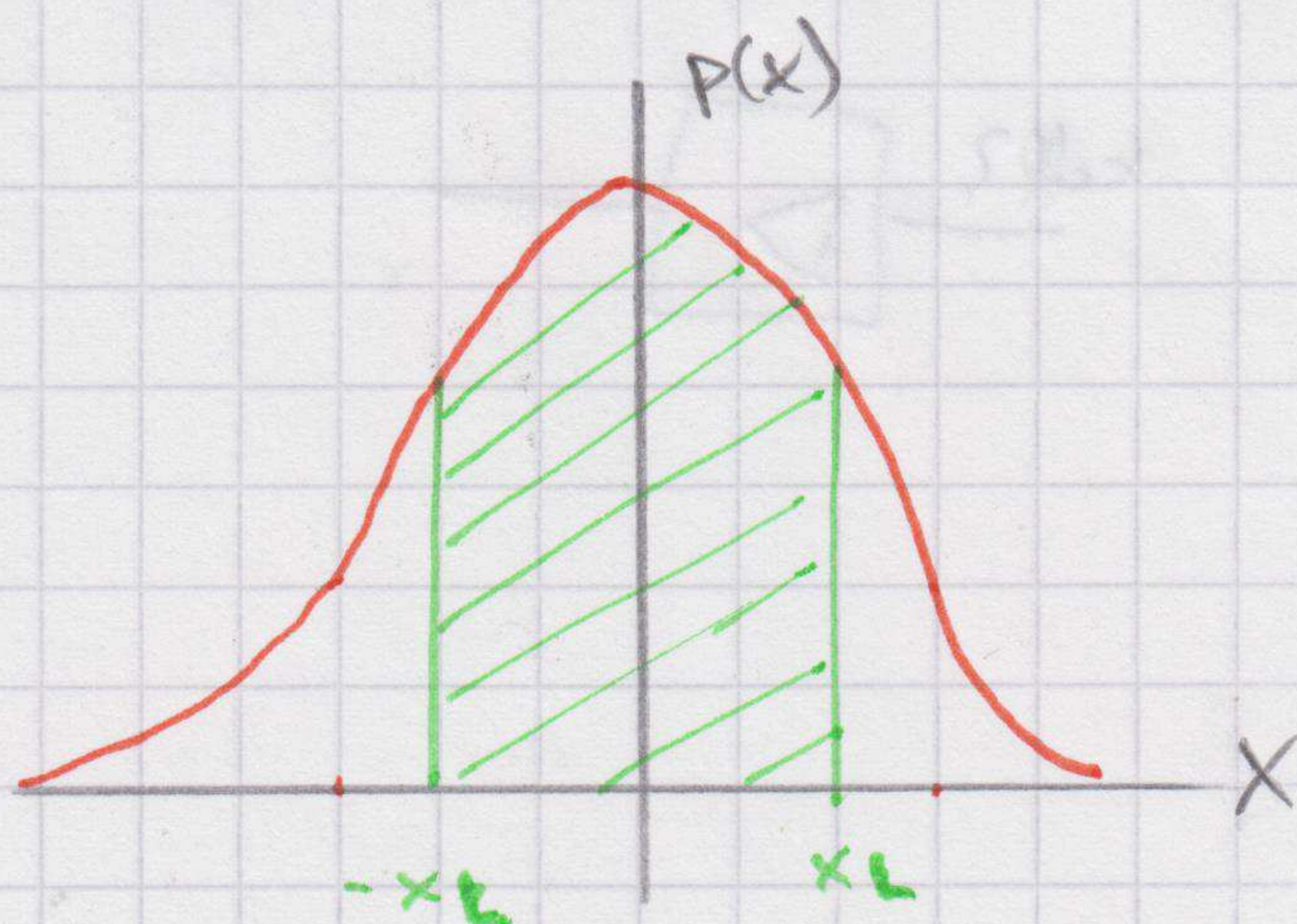
$$R(x) = \Phi(-x)$$

$$\Phi = \Phi_0 + \frac{1}{2}$$

$$\Phi_0(x) = -\Phi_0(-x)$$

Bereichswahrscheinlichkeit

Verstärker übersteuert nicht



$$P(-x_L < x < x_L) = P(|x| < x_L)$$

$$= \int_{-x_L}^{x_L} p(x) dx$$

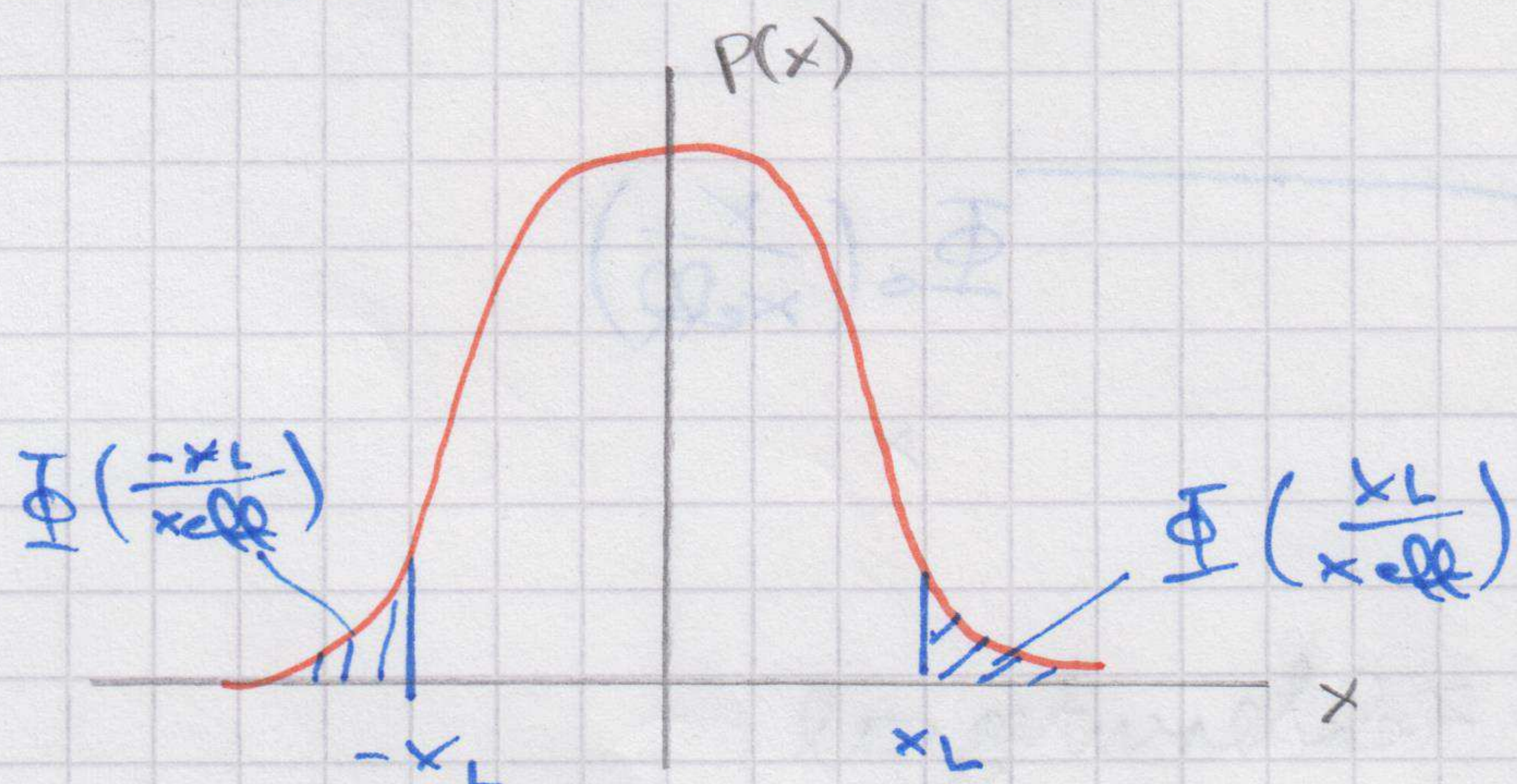
$$= \Phi\left(\frac{x_L}{x_{eff}}\right) - \Phi\left(\frac{-x_L}{x_{eff}}\right)$$

$$= \Phi_0\left(\frac{x_L}{x_{eff}}\right) - \Phi_0\left(\frac{-x_L}{x_{eff}}\right) = 2\Phi_0\left(\frac{x_L}{x_{eff}}\right)$$

<error function> = $\text{erf}\left(\frac{1}{\sqrt{2}} \cdot \frac{x_L}{x_{eff}}\right) \leftarrow (\text{Matheprogramme})$

Überschreitungswahrscheinlichkeit

Verstärker übersteuert



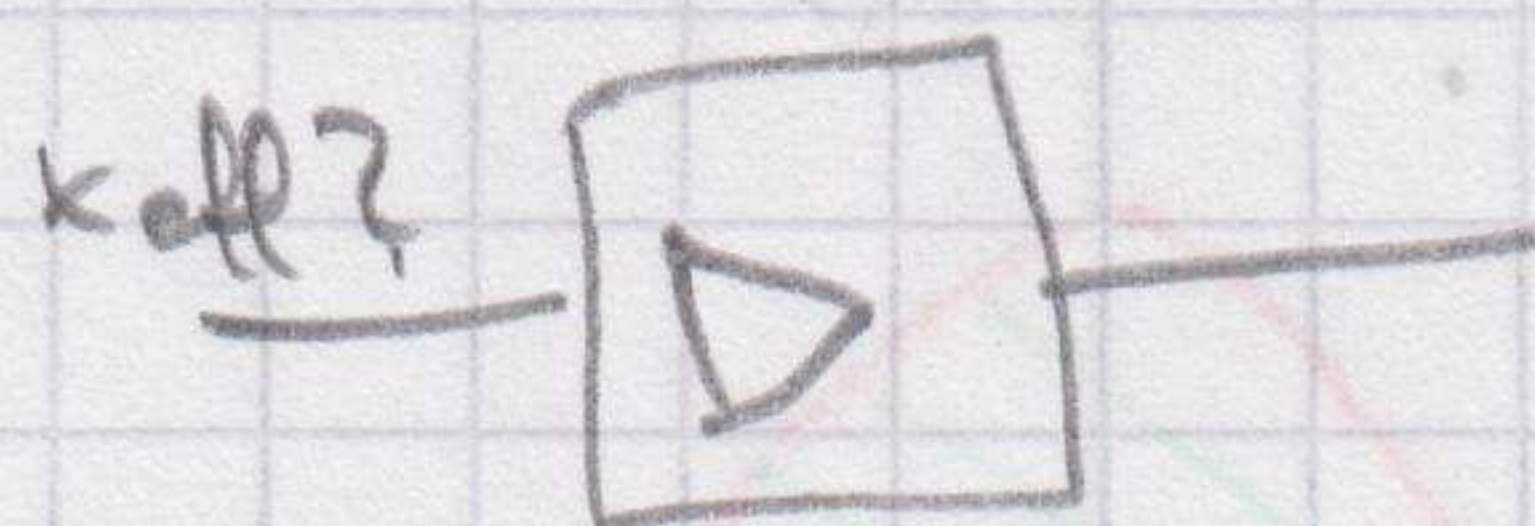
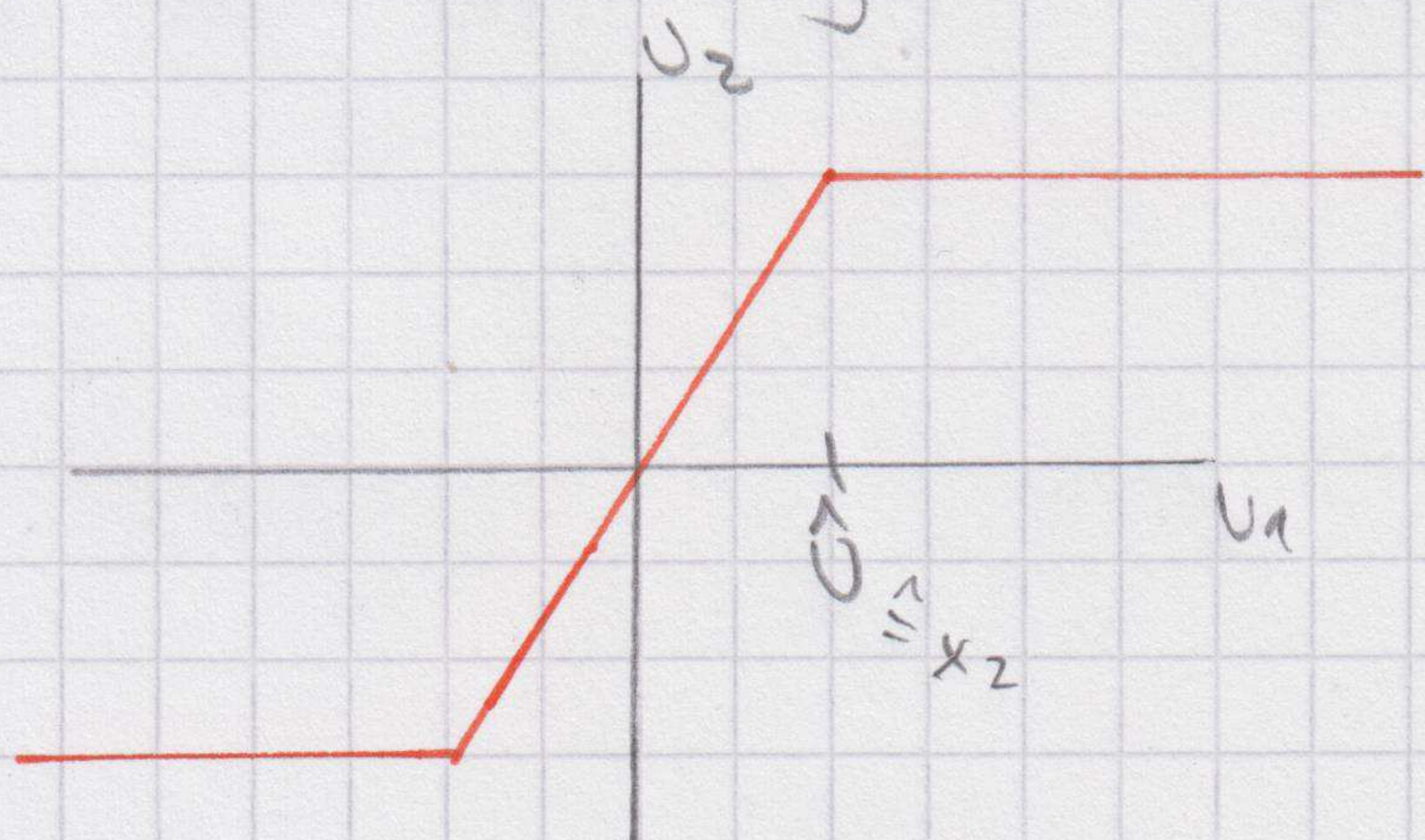
$$P(|x| > x_L) = \int_{-\infty}^{-x_L} p(x) dx + \int_{x_L}^{\infty} p(x) dx = 2\Phi\left(\frac{-x_L}{x_{eff}}\right)$$

$$= 1 - P(|x| < x_L) = 1 - 2\Phi_0\left(\frac{x_L}{x_{eff}}\right)$$

$$= \text{erfc}\left(\frac{1}{\sqrt{2}} \cdot \frac{x_L}{x_{eff}}\right)$$

$$\boxed{\text{erfc}(x) = 1 - \text{erf}(x)}$$

Bsp: Übersteuerung eines Verstärkers



Übersteuerung $\leq 2\%$

Überschreitungswahrscheinlichkeit

$$P(x > \hat{U}) = 2\% = 0,02$$

$$\text{erf}\left(\frac{1}{\sqrt{2}} \cdot \frac{\hat{U}}{U_{n,\text{eff}}}\right) = 0,02$$

$$\frac{1}{\sqrt{2}} \cdot \frac{\hat{U}}{U_{n,\text{eff}}} \approx 1,645$$

aus Diagramm

$$U_{n,\text{eff}} \approx \frac{\hat{U}}{1,645 \sqrt{2}} = \frac{2V}{1,645 \cdot \sqrt{2}} \approx 0,86V$$

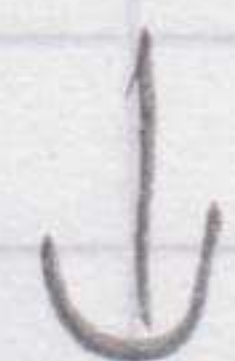
Mittelwerte

MW 1. Ordnung (Gleichanteil)

$$\overline{x(t)} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_T x(t) dt = x_{\text{eff}}$$

MW 2. Ordnung (Effektivwert)

$$\overline{x^2(t)} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_T x^2(t) dt = x_{\text{eff}}^2$$



Mittlere Gesamtleistung $P = \frac{x_{\text{eff}}^2}{R}$

↳ Ensemblemittelwert / Ensemble

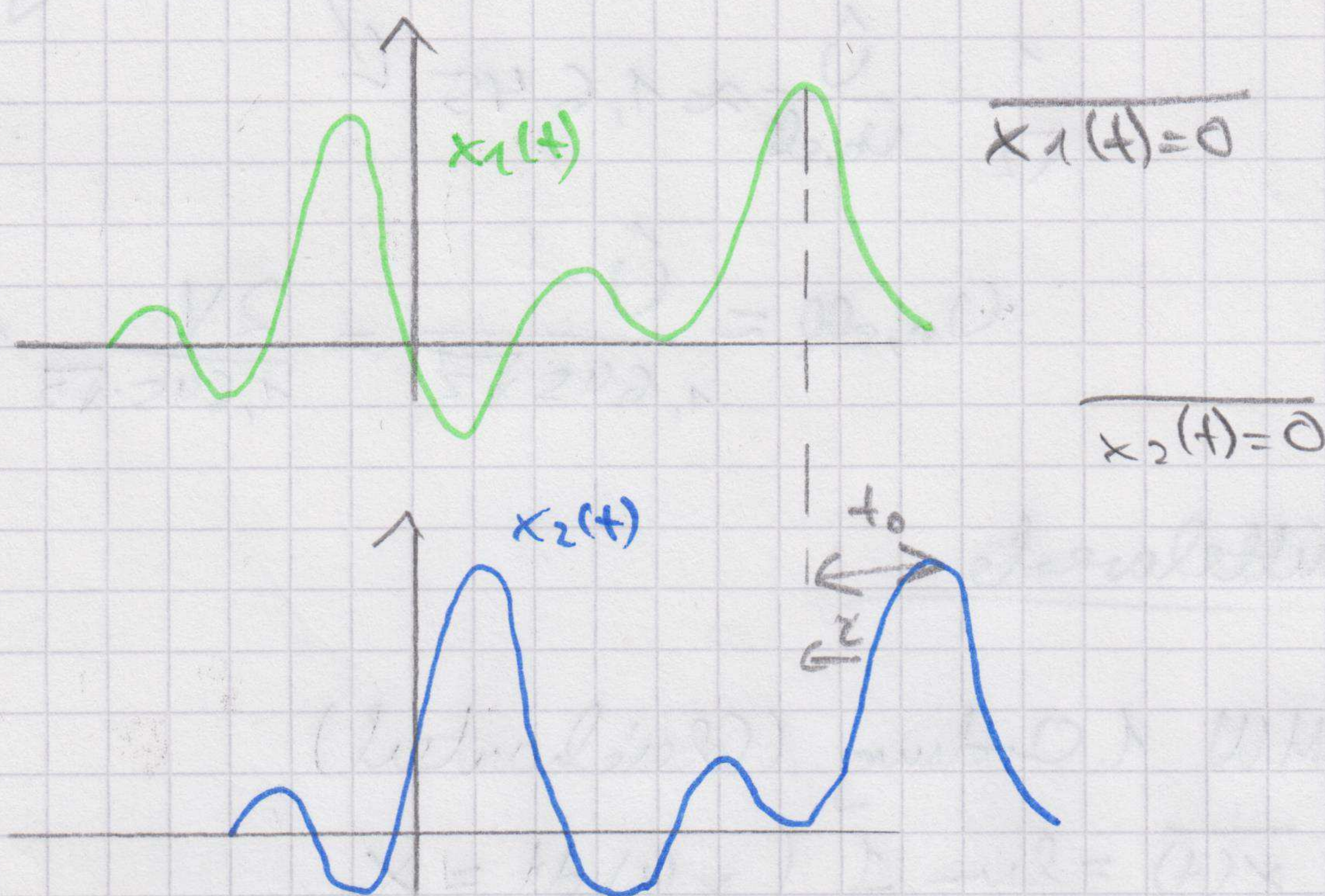
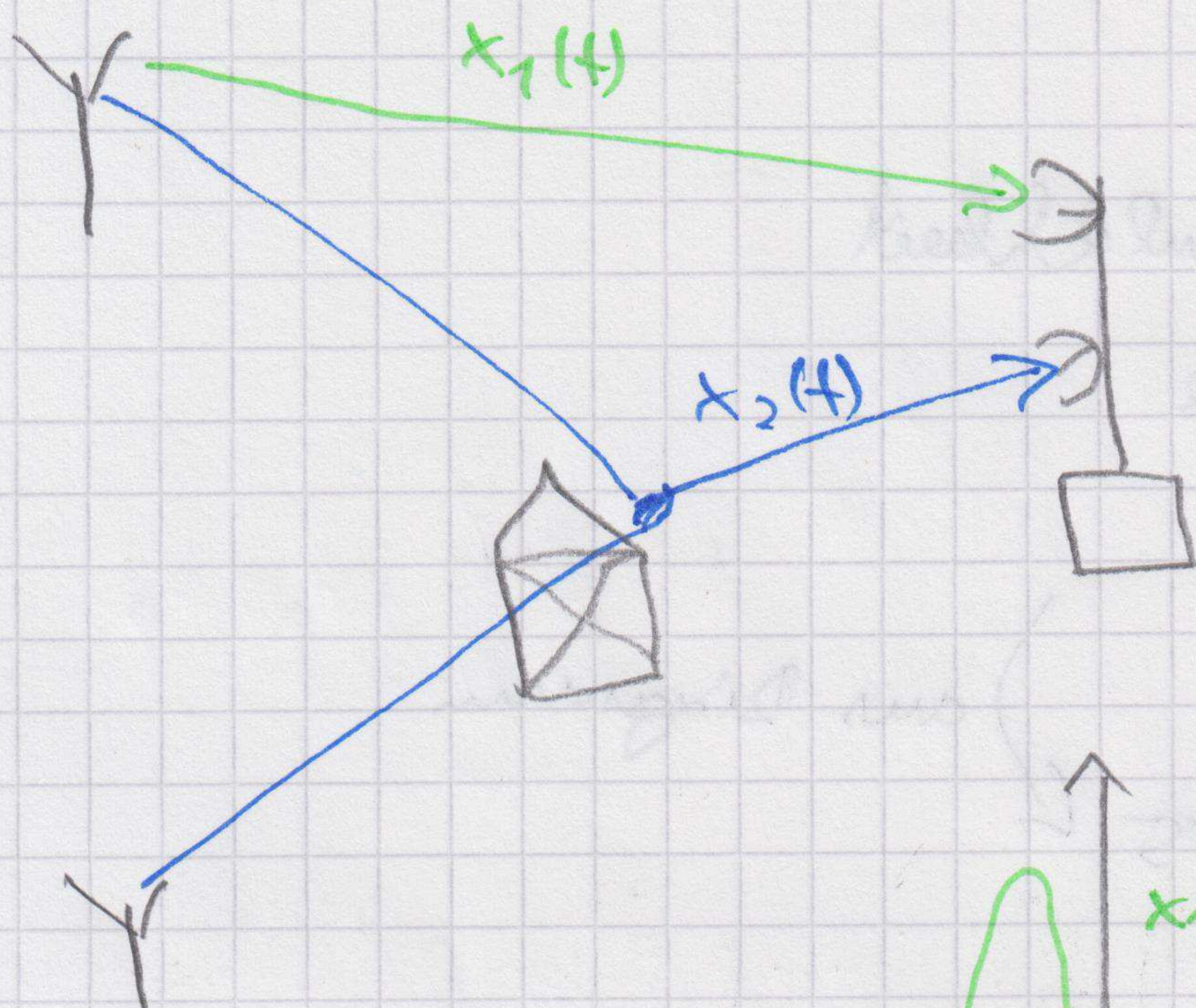
Moment 1. Ordnung

$$\widetilde{x(t)} = \int_{-\infty}^{\infty} x \cdot p(x) dx$$

(= $\overline{x(t)}$ weil ergodisch)

Moment 2. Ordnung

$$\widetilde{x^2(t)} = \int_{-\infty}^{\infty} x^2 p(x) dx$$

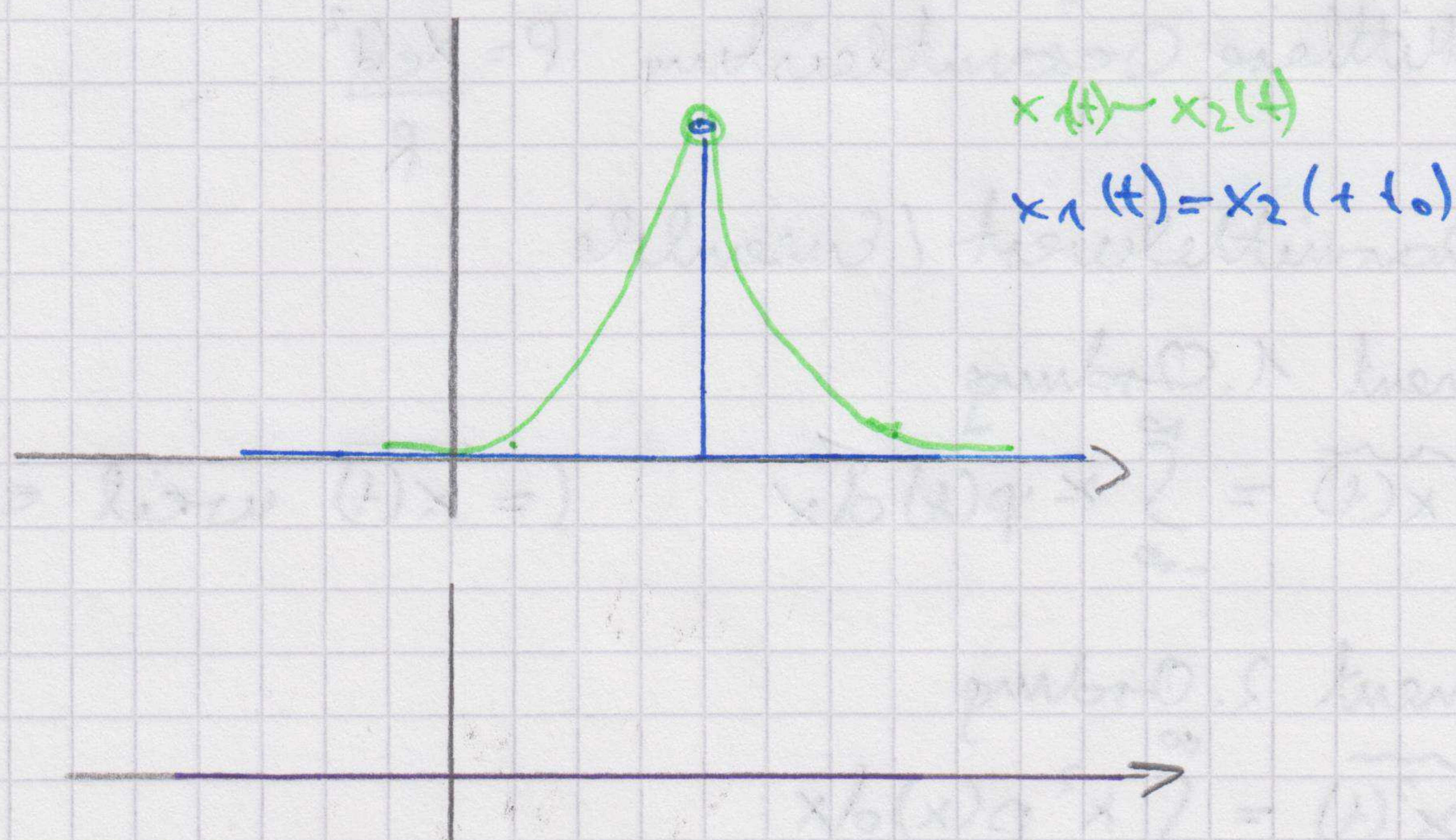


Bsp: $x_2(t) = x_1(t - t_0)$

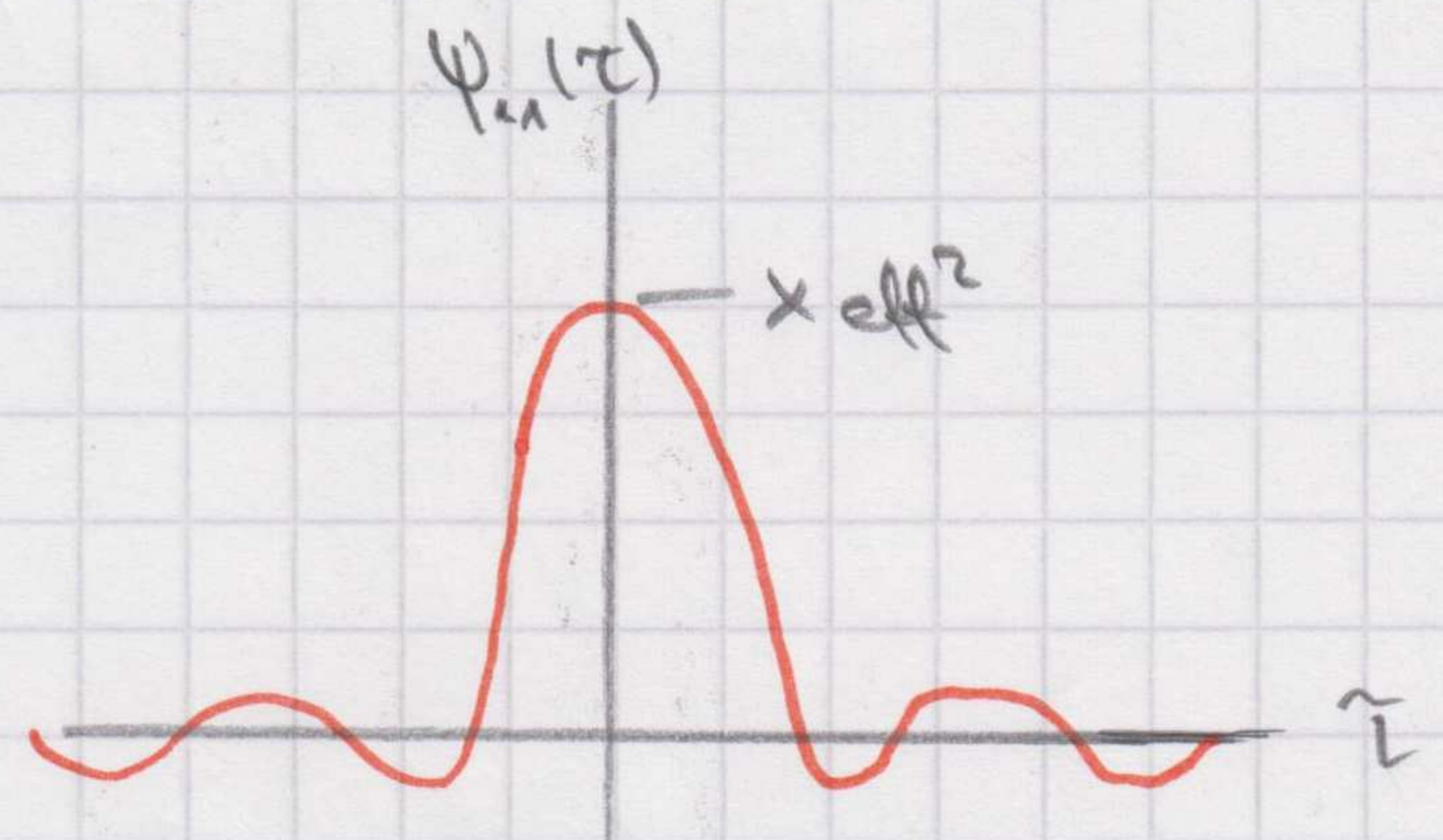
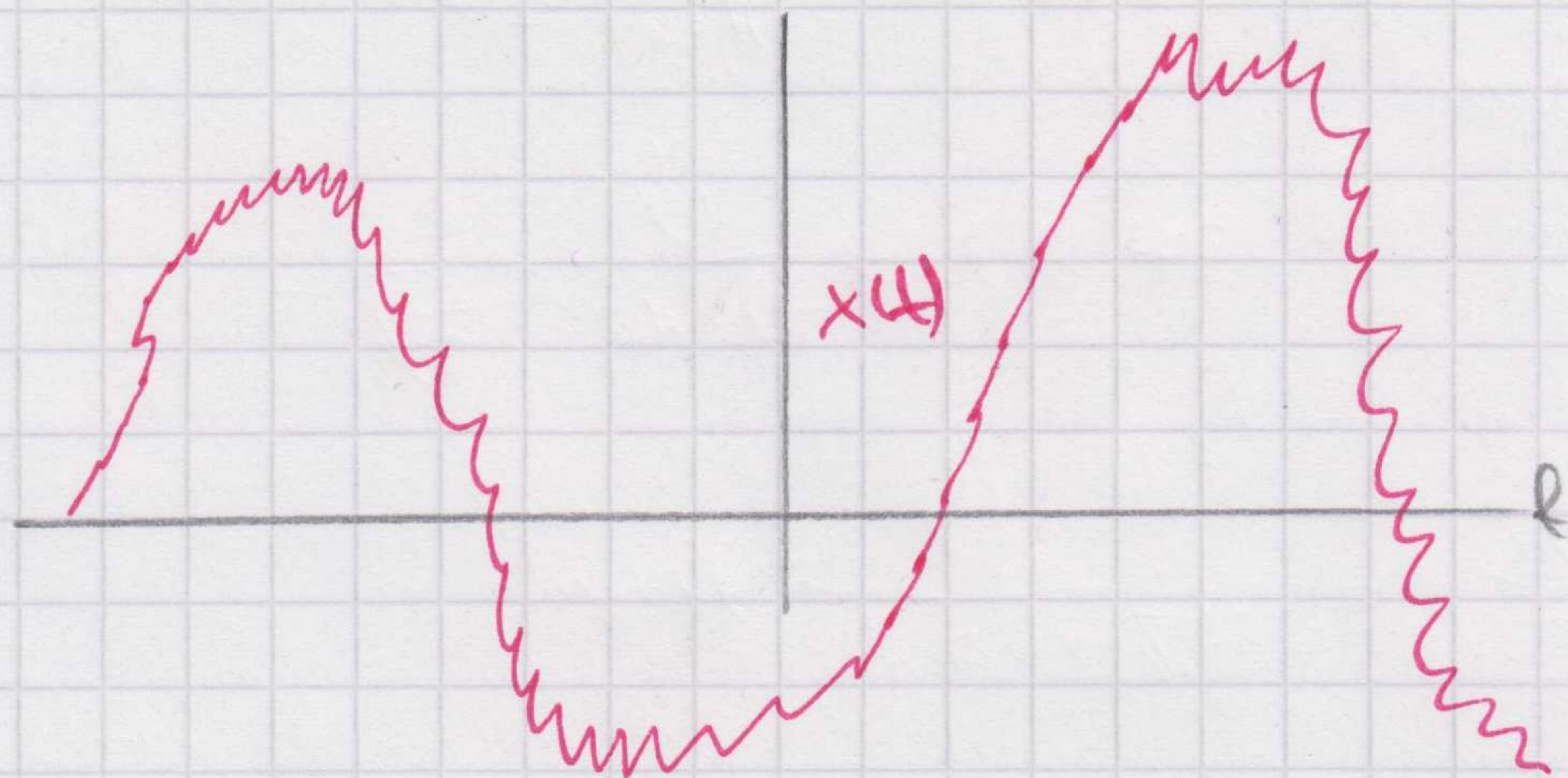
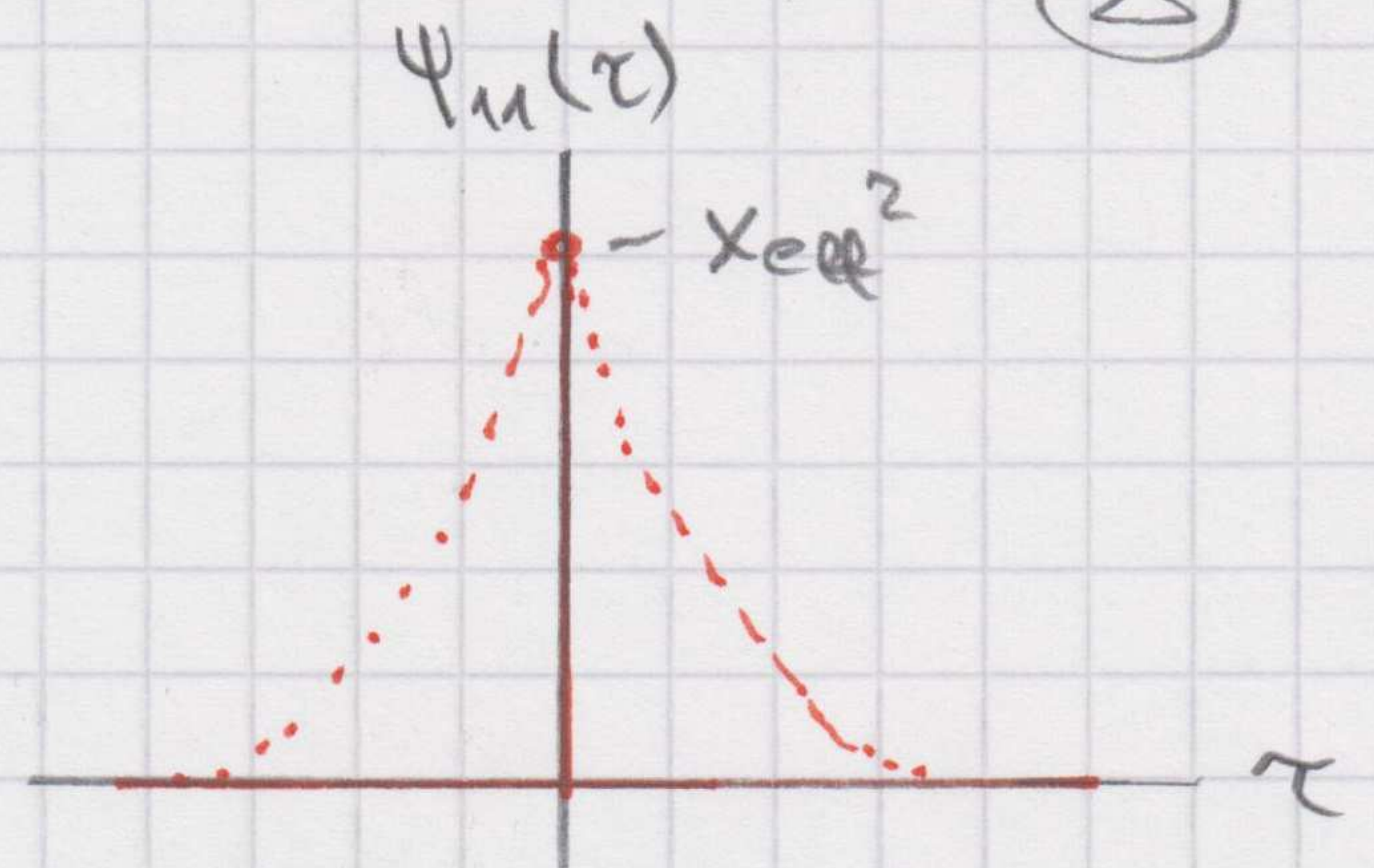
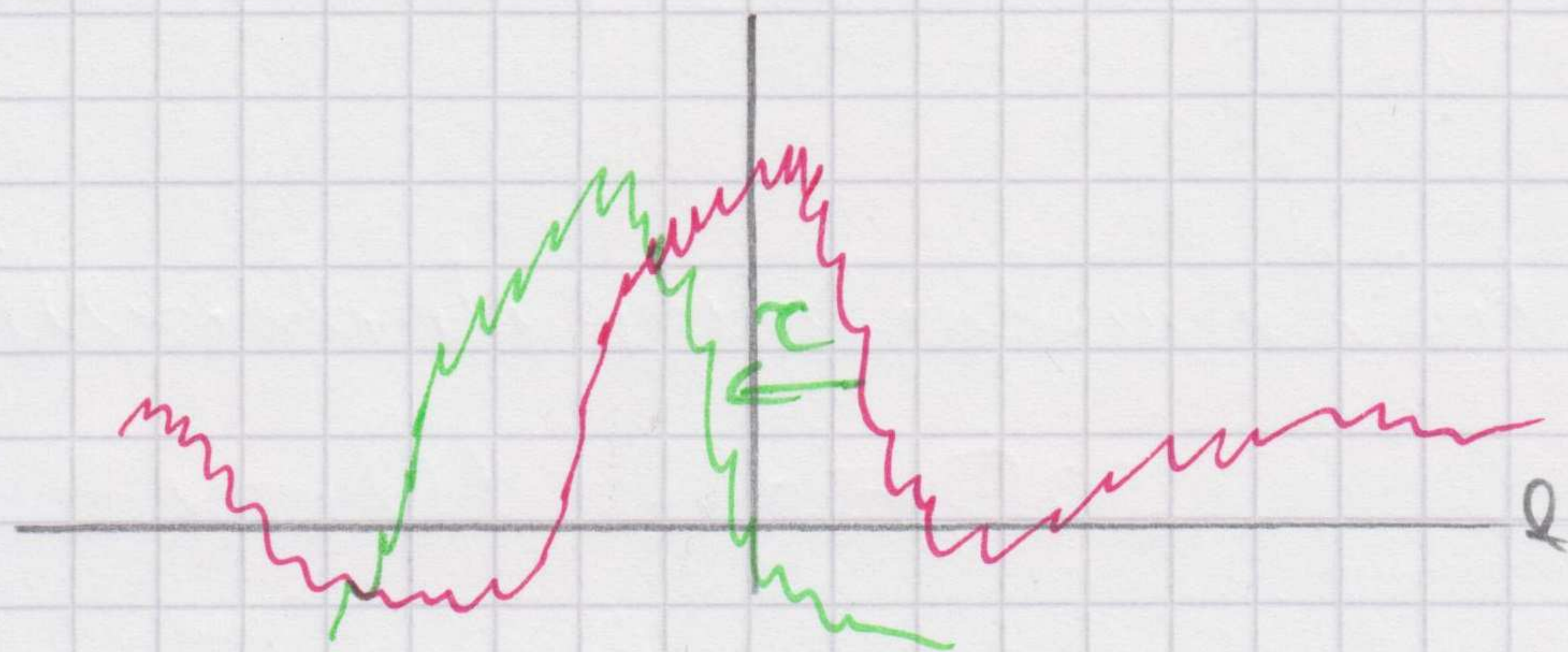
$$\varphi_{12}(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_T x_1(t) x_2(t + \tau) dt$$

Kreuzkorrelationsfunktion

$$x_1(t) x_2(t + \tau)$$



$\varphi_{12}(\tau) = 0$ (uncorrelated, orthogonal)



Autocorrelation function:

$$\Psi_{11}(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_T x(t) x(t+\tau) dt$$

$$= \overline{x(t) x(t+\tau)}$$

$$[\Psi_{11}(\tau)] = 1V^2$$

$$\Psi_{11}(0) = x_{eff}^2$$